Assessment Schedule – 2019

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

Q1	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\frac{dy}{dx}=\frac{1}{2}\left(3x^2-1\right)^{\frac{-1}{2}}.6x$ $=\frac{3x}{\sqrt{3x^2-1}}$	Correct derivative. Anything equivalent.		
(b)	$f'(t)=\frac{15}{3t-1}$ $f'(4)=\frac{15}{11}$ or 1.36	Correct solution with correct derivative.		
(c)	Quotient rule $\frac{dy}{dx}=\frac{\left(1+x^2\right)2e^{2x}-e^{2x}\left(2x\right)}{\left(1+x^2\right)^2}$ OR Product rule $\frac{dy}{dx}=e^{2x}\left(-2x\right)\left(1+x^2\right)^{-2}+\left(1+x^2\right)^{-1}\left(2e^{2x}\right)$ When $x=2, \frac{dy}{dx}=\frac{6e^4}{25}$ or 13.1	Correct derivative.	Correct solution with correct derivative.	
(d)	$\frac{dy}{dx}=3x^2e^x+x^3e^x$ $=x^2e^x\left(3+x\right)$ $\frac{dy}{dx}<0$ $\Rightarrow x^2e^x\left(3+x\right)<0$ $3+x<0$ $x<-3$	Correct derivative.	Correct solution with correct derivative.	
(e)	$\frac{dV}{dt}=\frac{dS}{dt}\times\frac{dr}{dS}\times\frac{dV}{dr}$ $S=4\pi r^2 \Rightarrow \frac{dS}{dr}=8\pi r$ $V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr}=4\pi r^2$ $\frac{dS}{dt}=0.4$ when $r=0.5$ $\frac{dV}{dt}=0.4\times\frac{1}{8\pi r}\times 4\pi r^2$ $=0.2r$ When $r=0.5, \frac{dV}{dt}=0.1\text{ m}^3/s$	Correct expressions for $\frac{dS}{dr}$ and $\frac{dV}{dr}$.	Correct expression for $\frac{dV}{dt}$. Anything equivalent. Line 5 is ok.	Correct solution with correct derivatives. Units not required.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=4(2x-5)^3.2$ $\frac{\mathrm{d}y}{\mathrm{d}x}=8(2x-5)^3$	Correct derivative.		
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\sec^2 2x$ $=\frac{2}{\cos^2 2x}$ At $x=\frac{\pi}{6}, \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{\cos^2\frac{\pi}{3}}=8$	Correct solution with correct derivative.		
(c)	$x=\frac{1}{(5-t)^2}=(5-t)^{-2}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=-2(5-t)^{-3}\times -1$ $=\frac{2}{(5-t)^3}$ $\frac{\mathrm{d}y}{\mathrm{d}t}=5-2t$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(5-2t)(5-t)^3}{2}$ At $t=2, \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1\times 3^3}{2}=13.5$	Correct expression for $\frac{\mathrm{d}x}{\mathrm{d}t}$.	Correct solution with correct derivatives shown.	
(d)	$\frac{\mathrm{d}\theta}{\mathrm{d}t}=0.01\text{ rad / s}$ $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}\theta}{\mathrm{d}t}\times\frac{\mathrm{d}h}{\mathrm{d}\theta}$ $\sin\theta=\frac{h}{22}$ $h=22\sin\theta$ $\frac{\mathrm{d}h}{\mathrm{d}\theta}=22\cos\theta$ $\therefore \frac{\mathrm{d}h}{\mathrm{d}t}=0.22\cos\theta$ $h=15 \Rightarrow \theta=\sin^{-1}\left(\frac{15}{22}\right)=0.75$ $\frac{\mathrm{d}h}{\mathrm{d}t}=0.22\cos(0.75)=0.16\text{ m s}^{-1}$	Correct expression for $\frac{\mathrm{d}h}{\mathrm{d}\theta}$.	Correct solution with correct derivative, $\frac{\mathrm{d}h}{\mathrm{d}\theta}$. Units not required.	

(e)	LHS $y = e^{\sin 2x}$ $\frac{dy}{dx} = e^{\sin 2x} \times 2\cos 2x$ $\frac{d^2y}{dx^2} = e^{\sin 2x} \times (-4\sin 2x) + e^{\sin 2x} \times (2\cos 2x)^2$ $u = \sin 2x$ $\frac{du}{dx} = 2\cos 2x$ $\frac{d^2u}{dx^2} = -4\sin 2x$ $y = e^u$ $\frac{dy}{du} = e^u$ $\frac{d^2y}{du^2} = e^u$ RHS $\frac{d^2y}{du^2} \times \left(\frac{du}{dx}\right)^2 + \frac{dy}{du} \times \frac{d^2u}{dx^2}$ $= e^u \times (2\cos 2x)^2 + e^u \times (-4\sin 2x)$ $= e^{\sin x} \times (2\cos 2x)^2 + e^{\sin x} \times (-4\sin 2x)$ Therefore LHS = RHS as required. $\frac{d^2y}{dx^2} = 4e^{\sin 2x}(\cos^2 2x - \sin 2x)$	Correct expression for $\frac{dy}{dx}$ or $\frac{du}{dx}$.	Correct expressions for $\frac{d^2y}{dx^2}$ in any equivalent form. Or correct RHS.	Complete proof. Accept in terms of x, y, and u.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$-4\sin^{-2}x\cos x$ OR $-4\operatorname{cosec}x\cot x$	Correct derivative.		
(b)(i) (ii)	1. $x=2, x>4$ 2. $x = -2, 1, 4$ Does not exist.	Two correct solutions i.e. TWO of (i) 1, (i) 2 and (ii).		
(c)	$A(x) = x\left(4-\sqrt{x}\right)$ $= 4x - x^{\frac{3}{2}}$ $A'(x) = 4 - \frac{3}{2}x^{\frac{1}{2}}$ Maximum area when $A'(x)=0$ $\frac{3}{2}\sqrt{x} = 4$ $\sqrt{x} = \frac{8}{3}$ $x = \frac{64}{9}$ Area $= \frac{64}{9}\left(4-\frac{8}{3}\right)$ $= \frac{64}{9}\times\frac{4}{3}$ $= \frac{256}{27} \quad \left(=9\frac{13}{27}\right)$ Accept 9.48	Correct expression for $A'(x)$.	Correct solution with correct derivative.	
(d)	$a(t) = 2e^{t} - 8e^{-t}$ $a(t) = 0$ $\Rightarrow 2e^{t} - 8e^{-t} = 0$ $2e^{t} = 8e^{-t}$ $e^{2t} = 4$ $2t = \ln 4$ $t = \frac{1}{2}\ln 4 \quad (= \ln 2 = 0.693)$	Correct derivative.	Correct solution with correct derivative.	

(e)	$y = 2\sqrt{36-x^2}$ $\frac{dy}{dx} = \left(36-x^2\right)^{\frac{-1}{2}}.-2x$ $= \frac{-2x}{\sqrt{36-x^2}}$ Gradient of tangent $= \frac{-2\sqrt{36-x^2}}{(8-x)}$ $= \frac{2\sqrt{36-x^2}}{x-8}$ $\therefore \frac{2\sqrt{36-x^2}}{x-8} = \frac{-2x}{\sqrt{36-x^2}}$ $2\left(36-x^2\right) = 16x-2x^2$ $72-2x^2 = 16x-2x^2$ $72 = 16x$ $x = 4.5$ Or alternatively: $y = \frac{-2x}{\sqrt{36-x^2}}(x-8)$ $y = \frac{-2x}{\sqrt{36-x^2}} + \frac{16x}{\sqrt{36-x^2}}$ Substituting for y: $2\sqrt{36-x^2} = \frac{-2x^2}{\sqrt{36-x^2}} + \frac{16x}{\sqrt{36-x^2}}$ $2\left(36-x^2\right) = -2x^2+16x$ $36-x^2 = -x^2+8x$ $36 = 8x$ $x = 4.5$	Correct $\frac{dy}{dx}$ of curve.	Correct $\frac{dy}{dx}$ of curve. AND Correct gradient of tangent. OR Correct equation of tangent involving expression for $\frac{dy}{dx}$.	Correct solution with correct derivatives.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 14	15 – 20	21 – 24