Assessment Schedule – 2018

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

Q1	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$6x^2 - 15\left(x^3+2\right)^{-4}.3x^2$	Correct derivative.		
(b)	$f'(x) = -9\sin 3x$ $f''(x) = -27\cos 3x$ $9f(x) + f''(x)$ $= 9\left(3\cos 3x\right) - 27\cos 3x$ $= 27\cos 3x - 27\cos 3x$ $= 0$	Correct proof with correct first and second derivatives.		
(c)	$y = \ln\left\|\sin^2 x\right\|$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sin x\cos x}{\sin^2 x}$ $= \frac{2\cos x}{\sin x}$ OR $y = \ln\left\|\sin^2 x\right\|$ $= 2\ln\left\|\sin x\right\|$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cos x}{\sin x}$ etc When $x = \frac{\pi}{6}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}$ $= 2\sqrt{3}$ $(= 3.4641)$	Correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$	Correct solution with correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$	

(d)	$\frac{dL}{dt} = 0.6 \text{ m s}^{-1}$ $L^2 = x^2 + 3^2$ $x = \sqrt{L^2 - 9}$ $\frac{dx}{dL} = \frac{1}{2}\left(L^2 - 9\right)^{\frac{-1}{2}} \cdot 2L$ $= \frac{L}{\sqrt{L^2 - 9}}$ $\frac{dx}{dt} = \frac{dL}{dt} \times \frac{dx}{dL}$ $= 0.6 \times \frac{L}{\sqrt{L^2 - 9}}$ When $L = 5.4$ $\frac{dx}{dt} = 0.6 \times \frac{5.4}{\sqrt{5.4^2 - 9}}$ $= 0.722 \text{ m s}^{-1}$	Correct expression for $\frac{dx}{dL}$ or $\frac{dL}{dx}$.	Correct solution with correct derivatives.	
(e)	$\frac{dx}{dt} = 3t^2 \quad \frac{dy}{dt} = 2t$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 2t \cdot \frac{1}{3t^2} = \frac{2}{3t}$ $\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dt} \times \frac{dt}{dx}$ $= \frac{-2}{3t^2} \times \frac{1}{3t^2} = \frac{-2}{9t^4}$ $\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^4} = \frac{\frac{-2}{9t^4}}{\left(\frac{2}{3t}\right)^4}$ $= \frac{-2}{9t^4} \times \frac{81t^4}{16}$ $= \frac{-9}{8}$ or -1.125	Correct $\frac{dy}{dx}$	Correct $\frac{d^2y}{dx^2}$	Correct solution with correct derivatives.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\frac{3}{2}x^{\frac{-1}{2}} - 5\operatorname{cosec}5x\cot 5x$	Correct derivative.		
(b)	$v(t) \quad \frac{6t + 3}{3t^2 + 3t + 1}$ $v(2) \quad \frac{15}{19}$ or 0.789 m s^{-1}	Correct solution with correct derivative.		
(c)(i) (ii) (iii)	5 –3, 1 (1) $1 < x < 3$ or $x > 7$ (2) 3 (3) 7	TWO out of five answers correct.	THREE out of five answers correct.	
(d)	$\frac{dy}{dx} = e^x\left(2x^2 - x - 1\right) + e^x(4x - 1)$ $= e^x\left(2x^2 + 3x - 2\right)$ $\frac{dy}{dx} = 0 \Rightarrow e^x\left(2x^2 + 3x - 2\right) = 0$ $\Rightarrow 2x^2 + 3x - 2 = 0$ $\Rightarrow (x + 2)(2x - 1) = 0$ $x = -2$ or $x = \frac{1}{2}$ Note $e^x = 0$ has no solutions since $e^x > 0 \forall x \varepsilon R$	Correct derivative.	Correct solution with correct derivative. Reference to $e^x = 0$ not required.	

(e)	$\frac{dV}{dt} = 150\ \text{cm}^3/\text{s}$ $\frac{dSA}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} \times \frac{dSA}{dr}$ $h = 2.5r$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{5}{6}\pi r^3$ $\frac{dV}{dr} = 2.5\pi r^2$ $SA = \pi r^2$ $\frac{dSA}{dr} = 2\pi r$ $\frac{dSA}{dt} = 150 \times \frac{1}{2.5\pi r^2} \times 2\pi r$ $= \frac{120}{r}$ When $h = 125$ cm, $r = 50$ cm $\frac{dSA}{dt} = \frac{120}{50} = 2.4\ \text{cm}^2/\text{s}$	Correct expression for $\frac{dV}{dr}$ in terms of one variable.	Correct expression for $\frac{dV}{dr}$ and $\frac{dSA}{dr}$ in terms of r, and an attempt to relate two (or more) derivatives.	Correct solution.

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No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\dfrac{\left(x^2+1\right).2e^{2x}-e^{2x}.2x}{\left(x^2+1\right)^2}$	Correct derivative.		
(b)	$\dfrac{dx}{dt}=10e^{2t}$ $\dfrac{dy}{dt}=10e^{5t}$ $\dfrac{dy}{dx}=\dfrac{10e^{5t}}{10e^{2t}}=\dfrac{e^{5t}}{e^{2t}}$ $t=0\Rightarrow\dfrac{dy}{dx}=1$	Correct solution with correct derivatives.		
(c)	$\text{Area}=\dfrac{1}{2}\cdot 2x\cdot\left(15-x^2\right)=15x-x^3$ $\dfrac{dA}{dx}=15-3x^2$ Max when $\dfrac{dA}{dx}=0$ $3\left(5-x^2\right)=0$ $x=\pm\sqrt{5}$ $y=10$ $\text{Area}=\dfrac{1}{2}\times 2\sqrt{5}\times 10$ $=10\sqrt{5}\ \ (=22.36)$	Correct $\dfrac{dA}{dx}$	Correct solution with correct derivative.	
(d)	$\dfrac{dy}{dx}=2x\cdot\ln x+x^2.\dfrac{1}{x}$ $\dfrac{dy}{dx}=2x\cdot\ln x+x$ $x=e\Rightarrow\dfrac{dy}{dx}=2e\cdot\ln e+e$ $=3e$ Equation of tangent: $y-y_1=m\left(x-x_1\right)$ $y-e^2=3e\left(x-e\right)$ $y-e^2=3ex-3e^2$ $y=3ex-2e^2$ $(y=8.155x-14.778)$	Correct expression for $\dfrac{dy}{dx}$	Correct solution with correct derivative. Accept any equivalent form.	

(e)	$w^2 = 5^2 + \left(5 - \frac{x}{2}\right)^2$ $w^2 = 25 + 25 - 5x + 0.25x^2$ $w^2 = 0.25x^2 - 5x + 50$ $w = \left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}$ $\text{Length} = x + 4w$ $= x + 4\left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}$ $\frac{dL}{dx} = 1 + 2\left(0.25x^2 - 5x + 50\right)^{\frac{-1}{2}} \times \left(0.5x - 5\right)$ $\frac{dL}{dx} = 1 + \frac{x - 10}{\left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}}$ For max/min $\frac{dL}{dx} = 0$ $\frac{x - 10}{\left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}} = -1$ $x - 10 = -1\left(0.25x^2 - 5x + 50\right)^{\frac{1}{2}}$ $\left(x - 10\right)^2 = 0.25x^2 - 5x + 50$ $x^2 - 20x + 100 = 0.25x^2 - 5x + 50$ $0.75x^2 - 15x + 50 = 0$ $x = 15.77$ not applicable $x = 4.23\text{ cm}$		Correct expression for $\frac{dL}{dx}$	Correct solution with correct derivative.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 12	13 – 18	19 – 24