Assessment Schedule – 2014

Calculus: Apply differentiation methods in solving problems (91578)

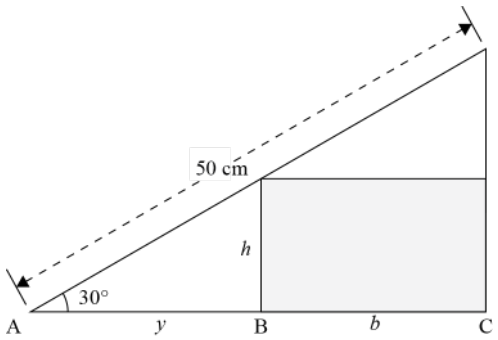
Evidence Statement

Q1	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$-15\sin(3x)$	A correct expression for the derivative.		
(b)	$\frac{dy}{dx} = 2(3x^2 - 5x)(6x - 5)$ At $x = 1$, $\frac{dy}{dx} = 2 \times {}^{-}2 \times 1 = {}^{-}4$ Gradient of normal $= \frac{1}{4}$ through (1,4)	A correct solution.		
(c)	$x = 2\sin t \quad y = \cos 2t$ $\frac{dx}{dt} = 2\cos t \quad \frac{dy}{dt} = -2\sin 2t$ $\frac{dy}{dx} = \frac{-2\sin 2t}{2\cos t}$ $= \frac{-2 \times 2\sin t\cos t}{2\cos t}$ $= -2\sin t$	Correct expressions for $\frac{dx}{dt}$ and $\frac{dy}{dt}$.	A correct solution.	
(d)	$y = \frac{4}{e^{2x-2}} + 8x = 4e^{-2x+2} + 8x$ $\frac{dy}{dx} = -8e^{-2x+2} + 8$ Parallel to x-axis $\Rightarrow \frac{dy}{dx} = 0$ $8e^{-2x-2} = 8$ $e^{-2x+2} = 1$ $-2x + 2 = 0$ $x = 1$	A correct expression for $\frac{dy}{dx}$.	A correct solution.	

(e)	$h^2 + r^2 = 400$ $h = \sqrt{400 - r^2}$ $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\sqrt{400 - r^2}$ $\frac{dV}{dr} = \frac{2}{3}\pi r\sqrt{400 - r^2} + \frac{1}{3}\pi r^2 . \frac{1}{2}\left(400 - r^2\right)^{\frac{-1}{2}} . -2r$ $\frac{dV}{dr} = \frac{\frac{2}{3}\pi r\left(400 - r^2\right) - \frac{1}{3}\pi r^3}{\sqrt{400 - r^2}}$ At maximum volume: $\frac{dV}{dR} = 0$ $2\left(400 - r^2\right) = r^2$ $3r^2 = 800$ $r = 16.3$ cm $V = 3225$ cm^3 Alternative working: $r^2 = 400 - h^2$ $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(400 - h^2\right)h$ $= \frac{\pi}{3}\left(400h - h^3\right)$ $\frac{dV}{dh} = \frac{\pi}{3}\left(400 - 3h^2\right)$ At maximum, $\frac{dV}{dh} = 0$ $400 - 3h^2 = 0$ $h^2 = \frac{400}{3}$ $h = \frac{20}{\sqrt{3}} = 11.547$ cm $V = 3225$ cm^3	Correct derivative for an incorrect but relevant expression for V.	A correct expression for $\frac{dV}{dr}$.	A correct solution. Units not required.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	ONE correct derivative	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$f'(x)=\dfrac{(2x-1)4e^{4x}-e^{4x}.2}{(2x-1)^2}$	A correct expression for the derivative.		
(b)	$y=8\ln(3x-2)$ $\dfrac{dy}{dx}=\dfrac{24}{(3x-2)}$ At $x=2$ $\dfrac{dy}{dx}=6$	A correct solution.		
(c)(i) (ii) (iii)	1. $-2, -1, 2$ 2. $x<-2$ 3. -2 4 3	2 correct answers	4 correct answers.	
(d)	$C=4v+\dfrac{1000000}{v}$ $\dfrac{dC}{dv}=4-\dfrac{1000000}{v^2}$ Minimum when $\dfrac{dC}{dv}=0$ $v^2=250000$ $v=500$ $C=4\times500+\dfrac{1000000}{500}=4000$	Correct value for v with correct derivative.	A correct solution. Units not required.	

(e)	 $\tan 30 = \frac{h}{y}$ $h = y\tan 30$ $\cos 30 = \frac{y+b}{50}$ $y + b = 50\cos 30$ $b = 50\cos 30 - y$ Area = base × height $A = \left(50\cos 30 - y\right)\left(y\tan 30\right)$ $= 50y\sin 30 - y^2\tan 30$ $= 25y - \frac{y^2}{\sqrt{3}}$ $\frac{dA}{dy} = 25 - \frac{2y}{\sqrt{3}}$ At $y = 20$ $\frac{dA}{dy} = 25 - \frac{40}{\sqrt{3}}$ $\frac{dA}{dt} = \frac{dA}{dy} \times \frac{dy}{dt}$ $= \left(25 - \frac{40}{\sqrt{3}}\right) \times 3$ $= 5.72 \text{ cm}^2 \text{ s}^{-1}$	Correct derivative for an incorrect but relevant expression for A.	A correct expressio n for $\frac{dA}{dy}$	A correct solution. Units not Required.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	ONE correct derivative	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$y=\left(\sqrt[3]{x^2+4x}\right)^2=\left(x^2+4x\right)^{\frac{2}{3}}$ $\frac{dy}{dx}=\frac{2}{3}\left(x^2+4x\right)^{\frac{-1}{3}}.\left(2x+4\right)$	A correct expression for the derivative.		
(b)	$y=x+\frac{32}{x^2}$ $\frac{dy}{dx}=1-\frac{64}{x^3}$ Stationary points when $\frac{dy}{dx}=0$ $\Rightarrow x^3=64$ $x=4$	A correct solution.		
(c)	$f(x)=5x-x\ln x$ $f'(x)=5-\ln x-\frac{x}{x}$ $=4-\ln x$ Increasing $\Rightarrow f'(x)>0$ $4-\ln x>0$ $\ln x<4$ $x<e^4$ $x<54.6$ But if $x\le 0$ then $\ln x$ is not defined, so $0<x<54.6$	A correct expression for the derivative.	A correct solution.	
(d)	$\frac{dh}{dt}=1.5\text{ m s}^{-1}$ $\tan\theta=\frac{h}{20}$ $h=20\tan\theta$ $\frac{dh}{d\theta}=20\sec^2\theta$ $\frac{d\theta}{dt}=\frac{d\theta}{dh}\times\frac{dh}{dt}$ $=\frac{1.5}{20\sec^2\theta}$ When $h=20$, $\theta=\frac{\pi}{4}$, $\sec^2\theta=2$ $\frac{d\theta}{dt}=\frac{1.5}{40}=0.0375$ radians s^{-1}	A correct expression for $\frac{dh}{d\theta}$	A correct solution. Units not required.	

(e)	$h = 40 - 2r$ $V = \pi r^2 h$ $= \pi r^2 (40 - 2r)$ $= 40\pi r^2 - 2\pi r^3$ $\frac{dV}{dr} = 80\pi r - 6\pi r^2$ $\frac{dV}{dr} = 0 \Rightarrow 80\pi r - 6\pi r^2 = 0$ $2\pi r(40 - 3r) = 0$ $r = \frac{40}{3}$ or 0 $r = \frac{40}{3}$ cm	Correct derivative for an incorrect but relevant expression for V.	A correct expression for $\frac{dV}{dr}$	A correct solution. Units not required.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	ONE correct derivative	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

	Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
Score range	0 – 7	8 – 12	13 – 20	21 – 24

Marking codes

Codes that may have been used in marking this examination paper have meaning as follows.

#	Hash where a candidate obtains a correct answer but continues with further, unnecessary, material that is incorrect but does not show a lack of understanding or a contradiction.
C	Consistency where a candidate has obtained an incorrect value within a question and subsequently uses that value.
NC	Non-consistency where a candidate has obtained an incorrect value or expression within a question and does not use that value or expression where it is subsequently required.
RAWW	Right answer, wrong working where a candidate presents a correct answer but the working or reasoning leading to it is irrelevant, incomplete or contains one or more errors.
R	Rounding error where a candidate produces a correct sequence of calculations, but the answer does not agree to 2 significant figures with the answer given in the assessment schedule as a result of rounding a number in the sequence of calculations.
mei	Minor error ignored where a candidate make a minor error and this has been ignored.
Two ans	Two answers given where a candidate writes two answers, one correct and the other incorrect, and neither has been deleted (the correct answer is not accepted).