Assessment Schedule – 2013

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

One	Expected Coverage	Achievement	Merit	Excellence
(a)	$\dfrac{dy}{dx} = \sec^2\left(x^2+1\right)\cdot 2x$	Correct derivative.		
(b)	$\dfrac{dy}{dx} = \dfrac{3-e^x}{3x-e^x}$ or no tangent exists At $x = 0$ gradient $= -2$	Correct solution with correct derivative shown.		
(c)	$\dfrac{dy}{dx} = -2xe^{6-x^2}$ $\dfrac{d^2y}{dx^2} = -2e^{6-x^2} + 4x^2e^{6-x^2}$ Point of inflection when $\dfrac{d^2y}{dx^2} = 0$ $\left(4x^2-2\right)e^{6-x^2} = 0$ $4x^2 - 2 = 0$ $x = \pm\dfrac{1}{\sqrt{2}}$	Correct $\dfrac{dy}{dx}$	Correct solution with correct first and second derivatives. $\pm$ not required, accept positive answer only.	
(d)	$\dfrac{dx}{dt} = 5\cos t \qquad \dfrac{dy}{dt} = 3\sec^2 t = \dfrac{3}{\cos^2 t}$ $\dfrac{dy}{dx} = \dfrac{3}{5\cos^3 t}$ At $t = \dfrac{\pi}{3}$, $\dfrac{dy}{dx} = \dfrac{24}{5}(= 4.8)$ $\therefore$ gradient of normal $= \dfrac{-5}{24}(= -0.2083)$	Correct $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$.	Correct solution including all correct derivatives.	

| (e) | $20 = 2\pi r^2 + 2\pi rh$
 $2\pi r(r+h) = 20$

 $h = \dfrac{10}{\pi r} - r$

 $V = \pi r^2 h = \pi r^2 \cdot \left(\dfrac{10}{\pi r} - r \right)$

 $V = 10r - \pi r^3$

 $\dfrac{dV}{dr} = 10 - 3\pi r^2$

 $\dfrac{dV}{dr} = 0 \implies r = \sqrt{\dfrac{10}{3\pi}}$ or $r = 1.03$ m

 OR $20 = \pi r^2 + 2\pi rh$

 $V = 10r - \dfrac{\pi r^3}{2}$

 $\dfrac{dV}{dr} = 10 - \dfrac{3\pi r^2}{2}$

 $r = \sqrt{\dfrac{20}{3\pi}} = 1.46$ | | Equation for volume in terms of 1 variable found, and differentiated correctly. | Problem solved including correct derivative. |

NØ = No response / no relevant evidence
N1 = ONE question demonstrating limited knowledge of differentiation techniques
N2 = ONE correct derivative
A3 = TWO of Achievement
A4 = THREE of Achievement
M5 = ONE of Merit
M6 = TWO of Merit
E7 = Excellence with minor errors ignored
E8 = Excellence correct

Two	Expected Coverage	Achievement	Merit	Excellence
(a)	$\dfrac{dy}{dx} = \dfrac{1}{3}\left(\pi - x^2\right)^{\frac{-2}{3}} \cdot -2x$ or $\dfrac{dy}{dx} = \dfrac{-2x}{3\left(\pi - x^2\right)^{\frac{2}{3}}}$	Correct derivative.		
(b)	$\dfrac{dy}{dx} = 3\left(x^3 - 2x\right)^2 \cdot \left(3x^2 - 2\right)$ At $x = 1$, $\dfrac{dy}{dx} = 3 \cdot (-1)^2 \cdot 1 = 3$ At $x = 1$, $y = -1$ $y + 1 = 3(x - 1)$ $y = 3x - 4$	Correct solution with correct derivative shown.		
(c)	$f'(x) = 1 - e^x + \dfrac{k}{x^2}$ $f'(x) = 0 \Rightarrow 1 - e^{-1} + k = 0$ $k = e^{-1} - 1$ Or $k = -0.632$	Correct derivative.	Correct value for k and correct derivative.	
(d)(i)	1. $x = 1$ 2. $x > 3$ 3. $-2, -1, 3$		THREE correct answers (out of 5).	
(ii)	-3			
(iii)	Does not exist.			
(e)	$A(\theta) = 64\sin\theta + 64\sin\theta\cos\theta$ OR $A(\theta) = 64\sin\theta + 32\sin 2\theta$ $A'(\theta) = 64\cos\theta + 64\cos^2\theta - 64\sin^2\theta$ OR $A'(\theta) = 64\cos\theta + 64\cos 2\theta$ $\qquad = 64\cos\theta + 64\cos^2\theta - 64(1 - \cos^2\theta)$ $\qquad = 64(2\cos^2\theta + \cos\theta - 1)$ Minimum when $A'(\theta) = 0$ $2\cos^2\theta + \cos\theta - 1 = 0$ $\left(2\cos\theta - 1\right)\left(\cos\theta + 1\right) = 0$ Or $\cos\theta = \dfrac{1}{2}$ or $\cos\theta = -1\,(\text{NO})$ $\theta = 60°$ or $\theta = \dfrac{\pi}{3}$		Correct derivative.	Correct solution with correct derivatives.

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Three	Expected Coverage	Achievement	Merit	Excellence
(a)	$$\dfrac{dy}{dx}=\dfrac{x^2\cdot\cos 2x\cdot 2-2x\sin 2x}{x^4}$$	Correct derivative.		
(b)	$f'(x)=1-16(x-2)^{-2}$ Turning point when $f'(x)=0$ $1=\dfrac{16}{(x-2)^2}$ $(x-2)^2=16$ $x=-2$ or $x=6$	Correct solution with correct derivative.		
(c)	$f'(x)=50-\left(30\ln 2x+30x\cdot\dfrac{1}{x}\right)$ $\qquad =20-30\ln 2x$ Maximum when $f'(x)=0$ $20=30\ln 2x$ $\dfrac{2}{3}=\ln 2x$ $x=\dfrac{e^{\frac{2}{3}}}{2}=0.974$	Correct derivative.	Correct solution with correct derivative.	
(d)	For the curve, $\dfrac{dy}{dx}=\dfrac{3t^2-3}{2t-1}$ Normal parallel to the y-axis means tangent parallel to the x-axis. $\Rightarrow\dfrac{dy}{dx}=0$ $3t^2=3$ $t=\pm 1$ $t=1\Rightarrow$ point $(0,-2)$ $t=-1\Rightarrow$ point $(2,2)$	Correct expression for $\dfrac{dy}{dx}$	Correct solution with correct derivative.	

(e)	$\dfrac{dV}{dt} = 300$ $A = 4\pi r^2 \quad \Rightarrow \dfrac{dA}{dr} = 8\pi r$ $V = \dfrac{4}{3}\pi r^3 \quad \Rightarrow \dfrac{dV}{dr} = 4\pi r^2$ $\dfrac{dA}{dt} = \dfrac{dV}{dt} \cdot \dfrac{dA}{dr} \cdot \dfrac{dr}{dV}$ $\qquad = \dfrac{2400\pi r}{4\pi r^2}$ $\qquad = \dfrac{600}{r}$ $A = 7500 \quad \Rightarrow 4\pi r^2 = 7500$ $\qquad r = \sqrt{\dfrac{7500}{4\pi}} = 24.43 \text{ cm}$ $\therefore \dfrac{dA}{dt} = \dfrac{600}{24.43} = 24.56 \text{ cm}^2\text{ s}^{-1}$	Correct expressions for $\dfrac{dV}{dr}$ and $\dfrac{dA}{dr}$	Correct expressions for $\dfrac{dV}{dr}$, $\dfrac{dA}{dr}$ and $\dfrac{dA}{dt}$	Correct solution along with correct expressions for $\dfrac{dV}{dr}$, $\dfrac{dA}{dr}$ and $\dfrac{dA}{dt}$

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Judgement Statement

	Not Achieved	**Achievement**	**Achievement with Merit**	**Achievement with Excellence**
Score range	0 – 8	9 – 13	14 – 20	21 – 24