Assessment Schedule – 2020

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$st = (2+3\mathrm{i})(3+k\mathrm{i})$ $= 6 + 2k\mathrm{i} + 9\mathrm{i} + 3k\mathrm{i}^2$ $= (6-3k) + (2k+9)\mathrm{i} = 21 - \mathrm{i}$ $k = -5$	Correct solution.		
(b)	$x^2 + 4rx + r = 0$ $(4r)^2 - 4\times 1\times r = 0$ $16r^2 - 4r = 0$ $4r(4r-1) = 0$ $r = 0$ or $r = \frac{1}{4}$	Correct solution.		
(c)	$2\sqrt{x} - 5 = \sqrt{4x - g}$ $4x - 20\sqrt{x} + 25 = 4x - g$ $25 + g = 20\sqrt{x}$ $x = \left(\frac{25+g}{20}\right)^2$	Correct squaring of both sides.	Correct solution.	
(d)	$\frac{k+k\mathrm{i}}{1-\mathrm{i}} + \frac{2k}{1+\mathrm{i}} = \frac{(k+k\mathrm{i})(1+\mathrm{i}) + 2k(1-\mathrm{i})}{(1-\mathrm{i})(1+\mathrm{i})}$ $= \frac{k + k\mathrm{i} + k\mathrm{i} + k\mathrm{i}^2 + 2k - 2k\mathrm{i}}{2}$ $= \frac{2k}{2} = k$	Correct expansion with common denominator (line 2).	Correct solution.	

(e)	$\frac{1+T^2}{2T}=\frac{1+\left(\frac{a-b\mathrm{i}}{a+b\mathrm{i}}\right)^2}{2\left(\frac{a-b\mathrm{i}}{a+b\mathrm{i}}\right)}$ $=\frac{\left(\frac{a+b\mathrm{i}}{a+b\mathrm{i}}\right)^2+\left(\frac{a-b\mathrm{i}}{a+b\mathrm{i}}\right)^2}{\frac{2(a-b\mathrm{i})}{(a+b\mathrm{i})}}$ $=\frac{(a+b\mathrm{i})^2+(a-b\mathrm{i})^2}{(a+b\mathrm{i})^2}\times\frac{(a+b\mathrm{i})}{2(a-b\mathrm{i})}$ $=\frac{(a+b\mathrm{i})^2+(a-b\mathrm{i})^2}{(a+b\mathrm{i})\times 2(a-b\mathrm{i})}$ $=\frac{a^2+2ab\mathrm{i}+b^2\mathrm{i}^2+a^2-2ab\mathrm{i}+b^2\mathrm{i}^2}{2\left(a^2+ab\mathrm{i}-ab\mathrm{i}-b^2\mathrm{i}^2\right)}$ $=\frac{2\left(a^2-b^2\right)}{2\left(a^2+b^2\right)}$ $=\frac{\left(a^2-b^2\right)}{\left(a^2+b^2\right)}$	This line, or equivalent.	This line, or equivalent.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$2\times 2^3+q\times 2^2-17\times 2-10=0$ $16+4q-34-10=0$ $q=7$	Correct solution.		
(b)	$\lvert 5+3k\mathrm{i}\rvert=13$ $25+9k^2=169$ $9k^2=144$ $k^2=16$ $k=\pm 4$	Correct solutions (both required). Can be done by inspection.		
(c)	$2z^3-15z^2+bz-30=0$ $z=3+\mathrm{i}$ a solution $\Rightarrow z=3-\mathrm{i}$ a solution $(z-3-\mathrm{i})(z-3+\mathrm{i})=z^2-6z+10$ $2z^3-15z^2+bz-30=(2z-3)(z^2-6z+10)$ Other solution is $z=\frac{3}{2}$ and $b=38$.	Other TWO solutions found. OR b found.	Other TWO solutions found. AND b found.	
(d)	$u=p+p\mathrm{i} \quad v=-q+q\mathrm{i}$ $\frac{u}{v}=\frac{p+p\mathrm{i}}{-q+q\mathrm{i}}\times\frac{-q-q\mathrm{i}}{-q-q\mathrm{i}}$ $=\frac{-pq-pq\mathrm{i}-pq\mathrm{i}-pq\mathrm{i}^2}{2q^2}$ $=\frac{-2pq\mathrm{i}}{2q^2}=\frac{-p\mathrm{i}}{q}$ $\arg\left(\frac{u}{v}\right)=\frac{-\pi}{2}$ OR $\arg(u)=\frac{\pi}{4} \quad \arg(v)=\frac{3\pi}{4}$ $\arg\left(\frac{u}{v}\right)=\frac{\pi}{4}-\frac{3\pi}{4}=\frac{-\pi}{2}$	3rd line with i^2 substituted. OR Correct $\arg(u)$ and $\arg(v)$.	Correct solution. Accept other correct expressions of argument.	
(e)	$\lvert z+\mathrm{i}\rvert^2+\lvert z-\mathrm{i}\rvert^2=10$ Let $z=x+y\mathrm{i}$ $\lvert x+y\mathrm{i}+\mathrm{i}\rvert^2+\lvert x+y\mathrm{i}-\mathrm{i}\rvert^2=10$ $x^2+(y+1)^2+x^2+(y-1)^2=10$ $2x^2+y^2+2y+1+y^2-2y+1=10$ $2x^2+2y^2=8$ $x^2+y^2=4$		Correct expanded expression (line 4).	Correct solution.

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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$6k^2\text{cis}\left(\frac{2\pi}{3}\right)$	Correct solution.		
(b)	$z=5-\text{i} \quad w=-2+3\text{i}$ $\lvert z\rvert^2=26 \quad \lvert w\rvert^2=13$ $\therefore \lvert z\rvert^2=2\lvert w\rvert^2$	Correct solution.		
(c)	$\frac{z\bar{z}}{z+\bar{z}}=\frac{(a+b\text{i})(a-b\text{i})}{a+b\text{i}+a-b\text{i}}$ $=\frac{a^2+b^2}{2a}$ which is real	Both numerator and denominator evaluated correctly.	Correct solution with a statement indicating real part only OR imaginary part = 0.	
(d)	$z^4=-16k^8=16k^8\text{cis}(\pi)$ $z_1=2k^2\text{cis}\left(\frac{\pi}{4}\right)$ $z_2=2k^2\text{cis}\left(\frac{3\pi}{4}\right)$ $z_3=2k^2\text{cis}\left(\frac{-3\pi}{4}\right)$ $z_4=2k^2\text{cis}\left(\frac{-\pi}{4}\right)$	z^4 written correctly in polar form and one correct solution, OR all arguments correct.	Four correct solutions. Accept equivalents in degrees.	

(e)	$\lvert u+v\rvert = \lvert u-v\rvert$ $u+v = a+bi+c+di$ $= a+c+(b+d)i$ $u-v = a+bi-c-di$ $= a-c+(b-d)i$ $\lvert u+v\rvert = \lvert u-v\rvert$ $\Rightarrow (a+c)^2+(b+d)^2 = (a-c)^2+(b-d)^2$ $a^2+2ac+c^2+b^2+2bd+d^2$ $= a^2-2ac+c^2+b^2-2bd+d^2$ $2ac+2bd = -2ac-2bd$ $4ac+4bd = 0$ $ac+bd = 0$ $\frac{u}{v} = \frac{(a+bi)}{(c+di)} \times \frac{(c-di)}{(c-di)}$ $= \frac{ac-adi+bci-bdi^2}{c^2+d^2}$ $= \frac{ac+bd+(bc-ad)i}{c^2+d^2}$ But $ac+bd=0$ $\therefore \frac{u}{v} = \frac{(bc-ad)i}{c^2+d^2}$ which is purely imaginary	Correct expression for $\lvert u+v\rvert = \lvert u-v\rvert$ (Line beginning with arrow.)	Correct simplified expression for $\lvert u+v\rvert = \lvert u-v\rvert$ $ac+bd=0$ or equivalent.	Correct solution.

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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 14	15 – 20	21 – 24