

Assessment Schedule – 2019

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

Q1	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$x = 2 \pm \sqrt{3}i$	Correct solution.		
(b)	$2(3)^3 - (3)^2 - 4(3) + p = 38$ $54 - 9 - 12 + p = 38$ $p = 5$	Correct solution.		
(c)	$\left \frac{u}{v} \right = 13$ $\left \frac{q+2i}{1-2i} \right = 13$ $\frac{\sqrt{q^2+4}}{\sqrt{5}} = 13$ $\frac{q^2+4}{5} = 169$ $q^2+4 = 845$ $q^2 = 841$ $q = \pm 29$	Third line.	Correct solution.	
(d)	<p>$z = 1 - 2i$ is one solution $\Rightarrow z = 1 + 2i$ is also a solution.</p> $(z - (1 - 2i))(z - (1 + 2i))$ $= (z - 1 + 2i)(z - 1 - 2i)$ $= z^2 - z - 2zi - z + 1 + 2i + 2iz - 2i - 4i^2$ $= z^2 - 2z + 5$ $\therefore (z^2 - 2z + 5)(az + b) = 2z^3 - 5z^2 + cz - 5$ $a = 2, b = -1$ $\therefore (z^2 - 2z + 5)(2z - 1) = 0$ <p>Other solution is $z = \frac{1}{2}$</p> $2z^3 - 5z^2 + 12z - 5 = 0$ $c = 12$	Other two solutions found OR c found.	Other two solutions found AND c found.	

<p>(e)</p> $\frac{1}{x+iy} - \frac{1}{1+i} = 1-2i$ $\frac{1}{x+iy} - \frac{1(1-i)}{(1+i)(1-i)} = 1-2i$ $\frac{1}{x+iy} - \frac{1-i}{2} = 1-2i$ $\frac{2}{x+iy} - 1+i = 2-4i$ $\frac{2}{x+iy} = 3-5i$ $\frac{x+iy}{2} = \frac{1}{3-5i}$ $\frac{x+iy}{2} = \frac{(3+5i)}{(3-5i)(3+5i)}$ $\frac{x+iy}{2} = \frac{3+5i}{34}$ $x+iy = \frac{3+5i}{17}$ $x = \frac{3}{17} \quad y = \frac{5}{17}$	<p>Correctly rationalises a term.</p>	<p>Correct reduction of expression to two terms.</p> <p>OR</p> <p>Equates real and imaginary parts.</p>	<p>Correct solution.</p>
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NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 2	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\bar{p} = 3 + i$ $\bar{p} - 3q = 3 + i - 3(-2 + 5i)$ $= 3 + i + 6 - 15i$ $= 9 - 14i$	Correct solution.		
(b)	$\frac{3}{4 - \sqrt{5}} = \frac{3}{4 - \sqrt{5}} \times \frac{4 + \sqrt{5}}{4 + \sqrt{5}}$ $= \frac{12 + 3\sqrt{5}}{16 - 5}$ $= \frac{12}{11} + \frac{3\sqrt{5}}{11}$	Correct solution.		
(c)	$z^4 = -16p^2i$ $= 16p^2 \operatorname{cis}\left(\frac{-\pi}{2}\right)$ $z = \left(16p^2 \operatorname{cis}\left(\frac{-\pi}{2}\right)\right)^{\frac{1}{4}}$ $= 2\sqrt{p} \operatorname{cis}\left(\frac{-\pi}{8}\right)$ <p>Solutions are $\frac{\pi}{2}$ apart.</p> $z_1 = 2\sqrt{p} \operatorname{cis}\left(\frac{-\pi}{8}\right)$ $z_2 = 2\sqrt{p} \operatorname{cis}\left(\frac{3\pi}{8}\right)$ $z_3 = 2\sqrt{p} \operatorname{cis}\left(\frac{7\pi}{8}\right)$ $z_4 = 2\sqrt{p} \operatorname{cis}\left(\frac{-5\pi}{8}\right) \text{ or equivalent.}$	One correct solution, or all arguments correct.	Four correct solutions.	
(d)	$z = \frac{(\sqrt{3} + mi)(1 - \sqrt{3}i)}{(1 + \sqrt{3}i)(1 - \sqrt{3}i)}$ $= \frac{\sqrt{3} - 3i + mi - \sqrt{3}mi^2}{4}$ $= \frac{\sqrt{3} + \sqrt{3}m + (m - 3)i}{4}$ <p>z is purely real $\Rightarrow m = 3$</p>	Correct expanded expression (without i^2).	Correct solution.	

<p>(e)</p>	$z = x + iy$ $ z = 1 \Rightarrow x^2 + y^2 = 1$ $\frac{1+z}{1-z} = \frac{1+(x+iy)}{1-(x+iy)}$ $= \frac{(1+x)+iy}{(1-x)-iy}$ $= \frac{(1+x)+iy}{(1-x)-iy} \times \frac{(1-x)+iy}{(1-x)+iy}$ $= \frac{(1+x)(1-x) + (1+x)iy + iy(1-x) + i^2y^2}{(1-x)^2 - i^2y^2}$ $= \frac{1-x^2 + iy + iyx + iy - iyx - y^2}{(1-x)^2 + y^2}$ $= \frac{1-(x^2 + y^2) + 2iy}{(1-x)^2 + y^2}$ $= \frac{2iy}{(1-x)^2 + y^2}$ <p>which is purely imaginary.</p>	<p>Real and imaginary part grouped (4th line)</p>	<p>Correct expansion (without i^2).</p>	<p>Correct solution.</p>
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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 3	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\frac{u}{v} = \frac{q^2 \operatorname{cis}\left(\frac{3\pi}{4}\right)}{q^3 \operatorname{cis}\left(\frac{\pi}{3}\right)}$ $= \frac{1}{q} \operatorname{cis}\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)$ $= \frac{1}{q} \operatorname{cis}\left(\frac{5\pi}{12}\right)$	Correct solution.		
(b)	$x = 1$ and $y = -1$	Correct solution.		
(c)	$2\sqrt{x-3} = w\sqrt{x}$ $4(x-3) = w^2x$ $4x - 12 = w^2x$ $4x - w^2x = 12$ $x(4 - w^2) = 12$ $x = \frac{12}{(4 - w^2)}$	Correct equation with terms in x gathered (line 4).	Correct solution.	
(d)	$\frac{u}{v} = \frac{1+pi}{5+3i} \times \frac{5-3i}{5-3i}$ $= \frac{5-3i+5pi-3pi^2}{34}$ $= \frac{5+3p}{34} + \frac{5p-3}{34}i$ $\arg\left(\frac{u}{v}\right) = \left(\frac{\pi}{4}\right)$ $\Rightarrow \operatorname{Re}\left(\frac{u}{v}\right) = \operatorname{Im}\left(\frac{u}{v}\right)$ $\Rightarrow 5+3p = 5p-3$ $8 = 2p$ $p = 4$	3rd line or equivalent with real and imaginary terms grouped.	Correct solution.	

(e)	$x^2 + 3kx + k^2 = 7x + 3k$ $x^2 + (3k - 7)x + (k^2 - 3k) = 0$ $b^2 - 4ac = (3k - 7)^2 - 4 \times 1 \times (k^2 - 3k)$ $= 9k^2 - 42k + 49 - 4k^2 + 12k$ $= 5k^2 - 30k + 49$ $= 5(k^2 - 6k) + 49$ $= 5[(k - 3)^2 - 9] + 49$ $= 5(k - 3)^2 + 4$ $5(k - 3)^2 + 4 > 0 \text{ for all values of } k.$ <p>\therefore Quadratic equation will always have 2 real solutions.</p>	Correct expression for $b^2 - 4ac$.	Complete the square correctly started (line 7) or equivalent.	Correct solution.
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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 19	20 – 24