

Assessment Schedule – 2018**Calculus: Apply the algebra of complex numbers in solving problems (91577)****Evidence Statement**

Q1	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	15	Correct solution.		
(b)	$m^4 \text{cis} \frac{11\pi}{15}$	Correct solution.		
(c)	$4 + 4\sqrt{x} + x = x + k$ $4\sqrt{x} = k - 4$ $\sqrt{x} = \frac{k - 4}{4}$ $x = \left(\frac{k - 4}{4}\right)^2$		Correct solution.	
(d)	$k(1 + x^2) = 3 - 8x - x^2$ $k + kx^2 - 3 + 8x + x^2 = 0$ $(k + 1)x^2 + 8x + k - 3 = 0$ <p>For one repeated solution</p> $\Delta = 64 - 4(k + 1)(k - 3) = 0$ $16 - (k + 1)(k - 3) = 0$ $16 - (k^2 - 2k - 3) = 0$ $-k^2 + 2k + 19 = 0$ $k^2 - 2k - 19 = 0$ $(k - 1)^2 - 20 = 0 \text{ or } k = \frac{2 \pm \sqrt{4 + 76}}{2}$ $k = 1 \pm \sqrt{20}$ <p>OR $k = 1 \pm 2\sqrt{5}$</p>	Correct expression for Δ .	Correct exact solutions.	
(e)	$\frac{z}{\bar{z}} = \frac{(a + bi)}{(a - bi)} \times \frac{(a + bi)}{(a + bi)}$ $= \frac{a^2 + 2abi - b^2}{a^2 + b^2}$ $= \frac{a^2 - b^2}{a^2 + b^2} + \frac{2ab}{a^2 + b^2}i$ $c^2 + d^2 = \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2 + \left(\frac{2ab}{a^2 + b^2}\right)^2$ $= \frac{a^4 - 2a^2b^2 + b^4}{a^4 + 2a^2b^2 + b^4} + \frac{4a^2b^2}{a^4 + 2a^2b^2 + b^4}$ $= \frac{a^4 + 2a^2b^2 + b^4}{a^4 + 2a^2b^2 + b^4}$ $= 1$	Correct expression for $\frac{z}{\bar{z}}$ with common denominator (2nd line).	Correct expression for $c^2 + d^2$ found (4th line).	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 2	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$w = 2 + i$ clearly shown on Argand diagram	Correct solution.		
(b)	$9 + 3\sqrt{7}$	Correct solution.		
(c)	$z_1 = 3 + i, z_2 = 3 - i$ $(z - 3 - i)(z - 3 + i) = z^2 - 6z + 10$ \therefore third factor is $(z - 4)$, so $z_3 = 4$ $(z - 4)(z^2 - 6z + 10) = z^3 - 10z^2 + 34z - 40$ $\therefore A = -10$	Correct values for z_2 and z_3 OR Correct value of A.	Correct values for z_2 and z_3 AND Correct value of A.	
(d)	$z = \frac{15}{(1-2i)} \times \frac{(1+2i)}{(1+2i)} - 2i$ $= \frac{15(1+2i)}{5} - 2i$ $= 3 + 6i - 2i$ $= 3 + 4i$ $\text{mod}(z) = 5$	Correct expression with real denominator.	Correct solution.	
(e)	$ x + yi - 8 = x + iy - 4 + 2i $ $ (x - 8) + yi = (x - 4) + (2 + y)i $ $\sqrt{(x - 8)^2 + y^2} = \sqrt{(x - 4)^2 + (2 + y)^2}$ $x^2 - 16x + 64 + y^2 = x^2 - 8x + 16 + y^2 + 4y + 4$ $-16x + 8x + 64 - 20 = 4y$ $-8x + 44 = 4y$ $y = -2x + 11$ $x = 3, y = 5$ $\therefore m = 5$	Equation equating moduli without absolute value signs (3rd line).	Equation for locus with squared terms cancelled (line 5).	Correct solution set out logically and clearly.

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Q 3	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$uv = (3 - 2i)(2 + bi)$ $= 6 + 3bi - 4i + 2b$ $= (6 + 2b) + (3b - 4)i$ $b = 4$	Correct solution.		
(b)	$(x - 3p)^2 - 5p^2 = 0$ $(x - 3p)^2 = 5p^2$ $(x - 3p) = \pm\sqrt{5}p$ $x = 3p \pm \sqrt{5}p$ <p>Or by quadratic formula:</p> $x = \frac{6p \pm \sqrt{36p^2 - 4 \times 1 \times 4p^2}}{2}$ $= \frac{6p \pm \sqrt{20p^2}}{2}$ $= 3p \pm \sqrt{5}p$	Correct solution.		
(c)	$z^3 = k^6 \operatorname{cis}\left(\frac{-\pi}{2}\right)$ $z_1 = k^2 \operatorname{cis}\left(\frac{-\pi}{6}\right)$ $z_2 = k^2 \operatorname{cis}\left(\frac{\pi}{2}\right)$ $z_3 = k^2 \operatorname{cis}\left(\frac{-5\pi}{6}\right)$	<p>One correct solution (or general solution).</p> <p>Allow equivalent arguments.</p>	<p>All 3 solutions correct.</p> <p>Allow equivalent arguments.</p>	
(d)	$\operatorname{Arg}(w) = \frac{\pi}{4} \Rightarrow w = x + xi$ $w \cdot \bar{w} = (x + xi)(x - xi)$ $= x^2 - x^2i + x^2i - x^2i^2$ $= 2x^2$ $ w \cdot \bar{w} = \sqrt{(2x^2)^2}$ $= 2x^2$ $2x^2 = 20$ $x^2 = 10$ $x = \sqrt{10}$ $w = \sqrt{10} + \sqrt{10}i$ <p>Accept $w = 3.16 + 3.16i$ or equivalent decimal approximations.</p>	Correct expression for $w \cdot \bar{w}$	Correct solution.	

<p>(e)</p> $\frac{\sqrt{x+k} + \sqrt{x-k}}{\sqrt{x+k} - \sqrt{x-k}} = 4$ $\frac{(\sqrt{x+k} + \sqrt{x-k})}{(\sqrt{x+k} - \sqrt{x-k})} \times \frac{(\sqrt{x+k} + \sqrt{x-k})}{(\sqrt{x+k} + \sqrt{x-k})} = 4$ $\frac{x+k+2\sqrt{x^2-k^2}+x-k}{x+k-(x-k)} = 4$ $\frac{2x+2\sqrt{x^2-k^2}}{2k} = 4$ $\frac{x+\sqrt{x^2-k^2}}{k} = 4$ $x+\sqrt{x^2-k^2} = 4k$ $\sqrt{x^2-k^2} = 4k-x$ $x^2-k^2 = 16k^2-8kx+x^2$ $-k^2 = 16k^2-8kx$ $8kx = 17k^2$ $x = \frac{17k}{8}$	<p>2nd line.</p>	<p>A correct expression with simplified rational denominator.</p> <p>(line 4)</p>	<p>Correct solution presented in a logical manner.</p>
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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 20	21 – 24