Assessment Schedule – 2017

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

Q 1	Evidence	Achievement	Merit	Excellence
(a)	$-1+9i$	Correct solution.		
(b)	$\frac{36}{5-\sqrt{7}}\times\frac{5+\sqrt{7}}{5+\sqrt{7}}$ $=\frac{180+36\sqrt{7}}{25-7}$ $=10+2\sqrt{7}$	Correct solution.		
(c)	$p\sqrt{x-2}-5\sqrt{x}=0$ $p\sqrt{x-2}=5\sqrt{x}$ $p^2(x-2)=25x$ $p^2x-2p^2=25x$ $p^2x-25x=2p^2$ $x(p^2-25)=2p^2$ $x=\frac{2p^2}{p^2-25}$	Correct expression without surds.	Correct solution.	
(d)	$z=-2+\mathrm{i}$ is also a solution $(z-(-2-\mathrm{i}))(z-(-2+\mathrm{i}))(z-\alpha)=0$ $((z+2)+\mathrm{i})((z+2)-\mathrm{i})(z-\alpha)=0$ $(z^2+4z+5)(z-\alpha)=0$ $\therefore \alpha=6$ $(z^2+4z+5)(z-6)=0$ $z^3-2z^2-19z-30=0$ $\therefore B=-19$ Other 2 solutions are $z=-2+\mathrm{i}$ and $z=6$	One of $B=-19$ Or $z=6$ Correct.	$B=-19$ and both other solutions correctly given.	
(e)	$\lvert z+2-7i\rvert=2\lvert z-10+2i\rvert$ Use $z=x+\mathrm{i}y$ $\lvert x+\mathrm{i}y+2-7\mathrm{i}\rvert=2\lvert x+\mathrm{i}y-10+2\mathrm{i}\rvert$ $\lvert(x+2)+(y-7)\mathrm{i}\rvert=2\lvert(x-10)+(y+2)\mathrm{i}\rvert$ $\sqrt{(x+2)^2+(y-7)^2}=2\sqrt{(x-10)^2+(y+2)^2}$ $(x+2)^2+(y-7)^2=4(x-10)^2+4(y+2)^2$ $x^2+4x+4+y^2-14y+49$ $=4x^2-80x+400+4y^2+16y+16$ $3x^2-84x+3y^2+30y+363=0$ $x^2-28x+y^2+10y+121=0$ $(x-14)^2+(y+5)^2=100$	Correctly separating the real and complex components on both sides of the equation.	Correct equation for x and y without i's or surds.	Correct final equation.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response, no relevant evidence	ONE partial solution	1u	2u	3u	1r	2r	1t with minor error(s)	1t

Q 2	Evidence	Achievement	Merit	Excellence
(a)	–11	Correct solution.		
(b)	$2k$	Correct solution.		
(c)	$w=\frac{zw}{z}=\frac{15-3\text{i}}{-2+3\text{i}}$ $=\frac{15-3\text{i}}{-2+3\text{i}}\times\frac{-2-3\text{i}}{-2-3\text{i}}$ $=\frac{-30-45\text{i}+6\text{i}-9}{4+9}$ $=-3-3\text{i}$ $\text{Arg}(w)=\frac{5\pi}{4}$ or $\frac{-3\pi}{4}$ or equivalent	Correct expression for w.	Correct expression for w and correct argument for w as an exact value.	
(d)	$z^4=\frac{m}{\sqrt{2}}+\frac{m}{\sqrt{2}}\text{i}$ $\quad z^4=m\,\text{cis}\frac{\pi}{4}$ $z=\left(m\,\text{cis}\frac{\pi}{4}\right)^{\frac{1}{4}}$ $z_1=m^{\frac{1}{4}}\text{cis}\frac{\pi}{16}$ $z_2=m^{\frac{1}{4}}\text{cis}\frac{9\pi}{16}$ $z_3=m^{\frac{1}{4}}\text{cis}\frac{17\pi}{16}$ or $z_3=m^{\frac{1}{4}}\text{cis}\frac{-15\pi}{16}$ $z_4=m^{\frac{1}{4}}\text{cis}\frac{25\pi}{16}$ or $z_4=m^{\frac{1}{4}}\text{cis}\frac{-7\pi}{16}$	One correct solution. OR 4 correct arguments.	All four solutions correct.	
(e)	$u=\frac{k+4\text{i}}{1+k\text{i}}$ $=\frac{k+4\text{i}}{1+k\text{i}}\times\frac{1-k\text{i}}{1-k\text{i}}$ $=\frac{k-k^2\text{i}+4\text{i}+4k}{1+k^2}$ $=\frac{5k}{1+k^2}+\left(\frac{4-k^2}{1+k^2}\right)\text{i}$ $\text{Im}(u)=0 \Rightarrow \frac{4-k^2}{1+k^2}=0$ $4-k^2=0$ $k=\pm2$	Correct expression for u with a rational denominator.	$\frac{4-k^2}{1+k^2}=0$	Correct values of k.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial Solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 3	Evidence	Achievement	Merit	Excellence
(a)	$p^2\text{cis}\left(\frac{5\pi}{24}\right)$	Correct solution.		
(b)	$x = 3 \pm \sqrt{5}\,\text{i}$	Correct solution.		
(c)	$A = 2$ $B = -6$ $C = 23$	2 correct values.	3 correct values.	
(d)	$\frac{8+x}{x} = \sqrt{3}$ $8 + x = \sqrt{3}x$ $8 = \sqrt{3}x - x$ $x\left(\sqrt{3}-1\right) = 8$ $x = \frac{8}{\sqrt{3}-1}$ $= \frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $= \frac{8\sqrt{3}+8}{2}$ $= 4\sqrt{3}+4$ Alternative method: $8 + x = \sqrt{3}x$ $x^2 + 16x + 64 = 3x^2$ $2x^2 - 16x - 64 = 0$ $x^2 - 8x - 32 = 0$ $(x-4)^2 - 48 = 0$ $(x-4)^2 = 48$ $x - 4 = \pm\sqrt{48}$ $x = 4 \pm \sqrt{48}$ $x = 4 \pm 4\sqrt{3}$ Check: $x = 4 - 4\sqrt{3}$ is not a solution. $\therefore x = 4 + 4\sqrt{3}$	Solution with non-rational denominator (line 5). Solution in form $x = 4 \pm \sqrt{48}$	Correct solution in the form $x = a + b\sqrt{3}$	

(e)	**LHS** $= \dfrac{z^2+1}{2z}$ $= \dfrac{\dfrac{(a+bi)^2}{(a-bi)^2}+1}{2\left(\dfrac{a+bi}{a-bi}\right)}$ $= \dfrac{\dfrac{(a+bi)^2+(a-bi)^2}{(a-bi)^2}}{2\left(\dfrac{a+bi}{a-bi}\right)}$ A $= \dfrac{\dfrac{a^2+2abi-b^2+a^2-2abi-b^2}{(a-bi)^2}}{2\left(\dfrac{a+bi}{a-bi}\right)}$ $= \dfrac{\dfrac{2(a^2-b^2)}{(a-bi)^2}}{2\left(\dfrac{a+bi}{a-bi}\right)}$ B $= \dfrac{(a^2-b^2)}{(a+bi)(a-bi)}$ $= \dfrac{a^2-b^2}{a^2+b^2}$ Alternative method: **LHS** $= \dfrac{\dfrac{(a+bi)^2}{(a-bi)^2}+1}{\dfrac{2(a+bi)}{(a-bi)}}$ $= \dfrac{(a+bi)^2}{(a-bi)^2}\times\dfrac{(a-bi)}{2(a+bi)}+\dfrac{(a-bi)}{2(a+bi)}$ A $= \dfrac{(a+bi)}{2(a-bi)}+\dfrac{(a-bi)}{2(a+bi)}$ B $= \dfrac{1}{2}\left(\dfrac{(a+bi)}{(a-bi)}+\dfrac{(a-bi)}{(a+bi)}\right)$ $= \dfrac{1}{2}\left(\dfrac{a^2+2ab-b^2+a^2-2ab-b^2}{(a^2+b^2)}\right)$ $= \dfrac{1}{2}\left(\dfrac{2a^2-2b^2}{(a^2+b^2)}\right)$ $= \dfrac{a^2-b^2}{a^2+b^2}$	Correct substitution and down to A.	Correct substitution and down to B.	Correct proof.

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No response; no relevant evidence.	ONE partial Solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 20	21 – 24