

**Assessment Schedule – 2016**

**Calculus: Apply the algebra of complex numbers in solving problems (91577)**

**Evidence Statement**

Q1	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$s$ clearly marked at (1, 2)	Correct solution.		
(b)	$A = -6$	Correct solution.		
(c)	$(5 - \sqrt{x})^2 = x - p$ $25 - 10\sqrt{x} + x = x - p$ $25 - 10\sqrt{x} = -p$ $10\sqrt{x} = p + 25$ $100x = (p + 25)^2$ $x = \frac{(p + 25)^2}{100}$	Correct 3rd line.	Correct solution with correct, logical working.	
(d)	$(1 + 2i)^2 + \frac{1 + 2i}{1 - 2i}$ $= -3 + 4i + \frac{(1 + 2i)(1 + 2i)}{(1 - 2i)(1 + 2i)}$ $= -3 + 4i + \frac{-3 + 4i}{5}$ $= -3\frac{3}{5} + 4\frac{4}{5}i$	Correct calculation of $w^2$ or $\frac{w}{w}$	Correct solution with working clearly shown.	
(e)	$ x + yi - 2 + 3i  =  x + yi - 1 $ $ x - 2 + (y + 3)i  =  x - 1 + iy $ $\sqrt{(x - 2)^2 + (y + 3)^2} = \sqrt{(x - 1)^2 + y^2}$ $(x - 2)^2 + (y + 3)^2 = (x - 1)^2 + y^2$ $x^2 - 4x + 4 + y^2 + 6y + 9 = x^2 - 2x + 1 + y^2$ $-4x + 13 + 6y = -2x + 1$ $6y = 2x - 12$ $y = \frac{1}{3}x - 2$ $\therefore \text{Gradient} = \frac{1}{3}$		Correct 4th line.	Correct gradient with correct working.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial Solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$x = 3 \pm \sqrt{3}i$	Correct solution.		
(b)	$(2 + 3i)(5 + mi)$ $= 10 + 2mi + 15i + 3mi^2$ $= 10 - 3m + (15 + 2m)i$ $= 22 + 7i$ $\therefore m = -4$	Correct solution.		
(c)	$z^3 = 8k^6 \text{cis}(\pi)$ $z_1 = 2k^2 \text{cis}\left(\frac{\pi}{3}\right)$ $z_2 = 2k^2 \text{cis}(\pi)$ $z_3 = 2k^2 \text{cis}\left(\frac{5\pi}{3}\right)$ OR $z_3 = 2k^2 \text{cis}\left(\frac{-\pi}{3}\right)$	1 correct solution.	3 correct solutions.	
(d)	$\left  \frac{4+2i}{1+i} \times \frac{1-i}{1-i} \right  = \left  \frac{4-4i+2i-2i^2}{1-i^2} \right $ $= \left  \frac{6-2i}{2} \right $ $=  3-i $ $= \sqrt{3^2 + (-1)^2}$ $= \sqrt{10}$	6 - 2i in numerator.	Correct solution with working clearly shown.	
(e)	$2\sqrt{2x+k} = x - 8$ $4(2x+k) = x^2 - 16x + 64$ $8x + 4k = x^2 - 16x + 64$ $x^2 - 24x + 64 - 4k = 0$ One solution $\Rightarrow 24^2 - 4 \times 1 \times (64 - 4k) = 0$ $576 - 256 + 16k = 0$ $320 + 16k = 0$ $k = -20$		Correct equation without $\sqrt{x}$ .	Correct solution.

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial Solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$10 - 5\sqrt{3}$	Correct solution.		
(b)	$\frac{2}{3} \operatorname{cis} \frac{\pi}{12}$	Correct exact solution.		
(c)	$z = 3 - 4i$ $z - 3 = 4i$ $z^2 - 6z + 9 = -16$ $z^2 - 6z + 25 = 0$ Other factor is $(z - 2)$ $(z - 2)(z^2 - 6z + 25) = z^3 - 8z^2 + 37z - 50$ $\therefore B = 37$	Finding $(z - 2)$ is a factor.	Correct solution.	
(d)	Let $u = a + bi$ and $v = c + di$ $uv = ac + adi + bci + dbi^2$ $= (ac - bd) + (ad + bc)i$ $\overline{u\overline{v}} = (ac - bd) - (ad + bc)i$ $\overline{u} = a - bi$ $\overline{v} = c - di$ $\overline{u} \cdot \overline{v} = (a - bi)(c - di)$ $= ac - adi - bci + bdi^2$ $= ac - bd - (ad + bc)i = \overline{uv}$	Correct expression for LHS or RHS.	Correct solution.	
(e)	Let $u = a + bi$ and $v = c + di$ $ u + v ^2 = (a + c)^2 + (b + d)^2$ $= a^2 + 2ac + c^2 + b^2 + 2bd + d^2$ $ u - v ^2 = (a - c)^2 + (b - d)^2$ $= a^2 - 2ac + c^2 + b^2 - 2bd + d^2$ $ u + v ^2 =  u - v ^2$ $\Rightarrow 2ac + 2bd = -2ac - 2bd$ $4ac + 4bd = 0$ $ac + bd = 0$ $u\overline{v} = (a + bi)(c - di)i$ $= ac - adi + bci - bdi^2$ $= (ac + bd) + (bc - ad)i$ But from above $ac + bd = 0$ $\therefore u\overline{v}$ is purely imaginary		Correct expressions for $ u + v ^2$ and $ u - v ^2$ Expanded.	Correct solution.

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**Cut Scores**

<b>Not Achieved</b>	<b>Achievement</b>	<b>Achievement with Merit</b>	<b>Achievement with Excellence</b>
0–8	9–14	15–20	21–24