

Assessment Schedule – 2014

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

Q1	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$p = \frac{1}{2}$	OR equivalent.		
(b)	$u^4 = \left[\sqrt{18} \text{cis} \frac{-\pi}{4} \right]^4$ $= 324 \text{cis}(-\pi)$ or $324 \text{cis}(\pi)$	$u^4 = (4.24 \text{cis}(-45^\circ))^4$ $= 324 \text{cis}(-180^\circ)$ OR $324 \text{cis}(180^\circ)$	OR equivalent.	
(c)	$x - 3 = \sqrt{33 - 4x}$ $(x - 3)^2 = 33 - 4x$ $x^2 - 2x - 24 = 0$ $x = 6$ OR $x = -4$ CHECK: $x = 6$ $6 = \sqrt{9} + 3$ TRUE CHECK: $x = -4$ $-4 = \sqrt{49} + 3$ FALSE Therefore there is one solution: $x = 6$	$x = 6$ and -4 both given as solutions. OR $x = 6$ CRO	$x = 6$ chosen after discarding $x = -4$	
(d)	$z^4 = -4k^2i = 4k^2 \text{cis} \left(\frac{-\pi}{2} \right)$ OR $4k^2 \text{cis}(-90^\circ)$ $z_1 = \sqrt{2k} \text{cis} \left(\frac{-\pi}{8} \right)$ OR $z_1 = \sqrt{2k} \text{cis} \left(\frac{15\pi}{8} \right)$ $z_2 = \sqrt{2k} \text{cis} \left(\frac{3\pi}{8} \right)$ OR $z_2 = \sqrt{2k} \text{cis} \left(\frac{-13\pi}{8} \right)$ $z_3 = \sqrt{2k} \text{cis} \left(\frac{7\pi}{8} \right)$ OR $z_3 = \sqrt{2k} \text{cis} \left(\frac{-9\pi}{8} \right)$ $z_4 = \sqrt{2k} \text{cis} \left(\frac{11\pi}{8} \right)$ or $z_4 = \sqrt{2k} \text{cis} \left(\frac{-5\pi}{8} \right)$	One correct solution. OR 4 correct arguments in degrees or radians.	All correct solutions given.	
(e)	Let $z = x + iy$ $ x + iy - 1 + 2i = x + iy + 1 $ $ (x - 1) + (y + 2)i = (x + 1) + yi $ $\sqrt{(x - 1)^2 + (y + 2)^2} = \sqrt{(x + 1)^2 + y^2}$ $(x - 1)^2 + (y + 2)^2 = (x + 1)^2 + y^2$ $x^2 - 2x + 1 + y^2 + 4y + 4 = x^2 + 2x + 1 + y^2$ $-2x + 4y + 5 = 2x + 1$ $4y - 4x + 4 = 0$ $y - x + 1 = 0$ or $y = x - 1$	Correct 3rd line.	Correct 5th line. $(x - 1)^2 + \dots$	Correct expression for locus.

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	1 partial solution	1u	2u	3u	1r	2r	1t with 1 minor error	1 t

Q2	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$x = 2 \pm 2\sqrt{3}i$ or $x = 2 \pm \sqrt{12}i$	Correct solution.		
(b)	$w = 2\text{cis}\frac{\pi}{3}$ $w^4 = 16\text{cis}\frac{4\pi}{3}$ OR $w^4 = 16\text{cis}(240^\circ)$ $w^4 = -8 - 8\sqrt{3}i$ OR $w^4 = -8 - 13.86i$	OR Equivalent rectangular form.		
(c)	$w_1 = 2 - 3i$ $w_2 = 2 + 3i$ $[w - (2 - 3i)][w - (2 + 3i)]$ $= w^2 - 4w + 13$ $\Rightarrow g(x) = (w^2 - 4w + 13)(kw + c)$ $\Rightarrow k = 3$ $c = -2$ Coefficient of w $-4c + 13k$ $= 47$ $\Rightarrow A = 47$ OR $A = \frac{94 - 141i}{2 - 3i}$ $w_3 = \frac{2}{3}$	$w^2 - 4w + 13$ OR $A = 47$ OR $w_2 = 2 + 3i$ and $w_3 = \frac{2}{3}$ OR CRO	Correct value of A and other 2 solutions correct. $A = 47$ $w_2 = 2 + 3i$ $w_3 = \frac{2}{3}$	
(d)(i) (ii)	Circle centre (3,4), radius 2 5	Correct locus drawn.	Correct locus drawn and correct maximum value.	
(e)	$\frac{1 + 3i}{p + qi} = \frac{p - qi + 3pi + 3qj}{p^2 + q^2}$ $= \frac{p + 3q + (3p - q)i}{p^2 + q^2}$ $\text{Arg}(z) = \frac{\pi}{4} \rightarrow \tan(\text{Arg}(z)) = 1$ $\text{Re}(z) = \text{Im}(z)$ if $\text{Arg}(z) = \frac{\pi}{4}$ $\tan\left(\frac{\pi}{4}\right) = \frac{3p - q}{p + 3q}$ $\frac{3p - q}{p + 3q} = 1$ $p - 2q = 0$	Correct expression without i^2 (1st line).	$\text{Arg}(z) = \frac{\pi}{4}$ interpreted to give a correct relationship such as $\tan(\text{Arg}(z)) = \frac{\text{Im}(z)}{\text{Re}(z)}$	Correct solution.

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	1 partial solution	1u	2u	3u	1r	2r	1t with 1 minor error	1 t

Q3	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$25 - 16\sqrt{3}$	Correct expression.		
(b)	$r = -8 - 5i$ marked correctly on diagram.	Correct point indicated on diagram.		
(c)	$px^2 - 4px + 1 = 0$ has no real solutions $\rightarrow b^2 - 4ac < 0$ $16p^2 - 4p < 0$ $4p(4p - 1) < 0$ $0 < p < \frac{1}{4}$	Correct inequation (2nd line). OR CRO	Correct solution.	
(d)	$\bar{z}^2 + \frac{1}{z^2} = (3 - 2i)^2 + \frac{1}{(3 + 2i)^2}$ $= 5 - 12i + \frac{1}{5 + 12i}$ $= 5 - \frac{5}{169} - 12\frac{12}{169}i \text{ OR } \frac{850}{169} - \frac{2040}{169}i$ $\text{OR } 5.03 - 12.07i$	Correct expansion (2nd line). OR CRO.	Correct expression.	
(e)(i)	Roots $\alpha, \beta, \gamma \Rightarrow (x - \alpha)(x - \beta)(x - \gamma) = 0$ $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\gamma + \beta\gamma + \alpha\beta)x - \alpha\beta\gamma = 0$ Since $ax^3 + bx^2 + cx + d = 0$ $x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0$ Comparing coefficients of x^2 : $\alpha + \beta + \gamma = \frac{-b}{a}$ of x : $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$ (ii) of constant term: $\alpha\beta\gamma = \frac{-d}{a}$ $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \alpha\beta\gamma(\alpha + \beta + \gamma)$ $= \frac{-d}{a} \times \frac{-b}{a}$ $= \frac{bd}{a^2}$	One correct proof.	3 correct proofs.	All 4 proofs correct.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	1 partial solution	1u	2u	3u	1r	2r	1t with 1 minor error	1 t

Cut Scores

	Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
Score range	0 – 6	7 – 12	13 – 19	20 – 24