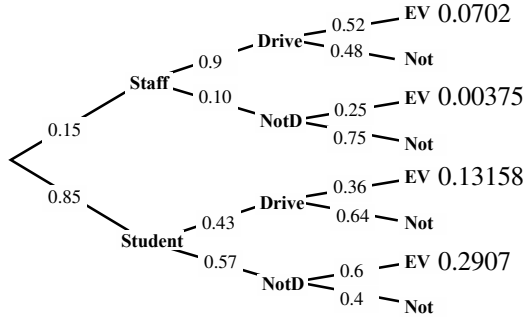
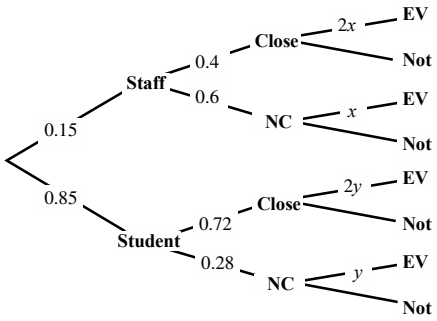
Assessment Schedule – 2020

Mathematics and Statistics: Apply probability methods in solving problems (91267)

Q ONE	Evidence	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
(a)(i)	P(Staff and Drove) = 0.15 × 0.9 = 0.135.	Probability correct. Tree not required.		
(ii)	P(Student and ND and would not like EV) = 0.85 × 0.57 × 0.4 = 0.1938	Probability correct.		
(iii)	P(Student would like EV) = p(Student, D, EV) + p(Student, ND, EV) = 0.85 × 0.43 × 0.36 + 0.85 × 0.57 × 0.6 = 0.13158 + 0.2907 = 0.42228 P(Student drove if want EV) = $\frac{0.1316}{0.4223}$ = 0.3116 For students, P(would like EV) = p(D, EV) + p(ND, EV) = 0.43 × 0.36 + 0.57 × 0.6 = 0.1548 + 0.3420 = 0.4968 P(Student drove if want EV) = $\frac{0.1548}{0.4968}$ = 0.3116	Either numerator or denominator correctly found. Allow consistency with their clearly drawn tree. OR CAO	Correct or consistent probability.	
(iv)	 P(want EV) = 0.15 × 0.9 × 0.52 + 0.15 × 0.1 × 0.25 + 0.85 × 0.43 × 0.36 + 0.85 × 0.57 × 0.6 = 0.0702 + 0.00375 + 0.13158 + 0.2907 (accept) = 0.4962 = 49.6% (accept decimal)	One new probability found OR CAO	Correct or consistent probability (all 4 added). Accept working on the tree.	

(b)	Need to define x and y. For staff who want to own EV: $0.4 \times 2x + 0.6x = 0.49$ $1.4x = 0.49$ so $x = 0.35$ For students who want to own EV: $0.72 \times 2y + 0.28y = 0.43$ $1.72y = 0.43$ so $y = 0.25$ P(want EV if Not Close) = P(Staff, NC, want EV) + P(Student, NC, want EV) $0.15 \times 0.6 \times 0.35 + 0.85 \times 0.28 \times 0.25$ $= 0.0315 + 0.0595 = 0.091$ P(want EV if living Close) = P(Staff, close, want EV) +P(Student, close, want EV) $= 0.15 \times 0.4 \times 0.7 + 0.85 \times 0.72 \times 0.5$ $= 0.042 + 0.306 = 0.348$ Prob that people wanting to own EV if living close is 0.348 which is more likely [or 3.8 times $\left(\frac{0.348}{0.091}\right)$ as likely] than prob that people wanting to own an EV if not living close (0.091).  	Tree diagram set up correctly with x and $2x$ or y and $2y$. OR CAO for x or y probabilities by trial and error.	Either x (prob Staff NC who want EV) or y (prob Student NC who want EV) found.	T1: correct x and y probabilities found OR Comparison of consistent probabilities for staff and students of wanting EV if Close and EV if NC from incorrect** values of x or y using relative risk or simple difference, with interpretation. ** as long as the doubling concept is clear on the tree, and the "x" and "y" are different. T2: Comparison of correct probabilities of wanting EV if Close and EV if NC using relative risk or simple difference, with interpretation.

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1 of u	2 of u	3 of u	1 of r	2 of r	T1	T2

Q TWO	Evidence	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
(a)(i)	$P(150 < x < 165) = P(-1.071 < Z < 0)$ $= 0.3580$	Correct probability.		
(ii)	$P(x > 172) = P(Z > 0.5) = 0.3085$	Correct probability.		
(iii)	Inverse normal $P(x > k) = 0.90$ $k = 147.06$ 90% of battery charges have a minimum distance of 147 km. Geoff is satisfied if he goes more than 147 km (147.1 or 147.06) on one full battery charge.	CAO OR Evidence of ±1.281	Correct minimum value obtained with working and / or diagram.	
(b)	$P(x < 265) = 0.2$ $P(Z < z) = 0.2 \quad z = -0.8416$ $0.8416 = \frac{(265 \quad)}{14}$ $\mu = 276.8$ km	CAO OR z-value of ±0.8416 found.	Correct z-value used but mean is incorrect	Correct mean found.

(c)	Comparison of the normal distribution model in claim with the sample distribution of test drives. Possible valid comments about <u>similarities</u>: **Centre** C1: Means are similar (model 280 vs 278.33 (n = 69) or 274.4 (n = 70) from data using frequencies of midpoints) or discusses mean likely about 280. C2: 47% of data is below 280, so median must be close to (but above) 280, suggesting a ND and claim could be valid. C3: Could calculate mean using b) or similar method using inverse normal and compare. About <u>differences</u>: **Spread** V1: Data has a range of 100, so approximate standard deviation of 17, which is larger than the model std dev of 14. V2: For example, Sd=14 → ±3sd approx range = 238 – 322 which is less than the experimental range. V3: For example, p(X < 250) = 0.016 (ND) but much higher $\frac{7}{70}=0.1$ in data, showing more data on the left than ND so the spread must be greater. **Shape** S1: A normal distribution is symmetrically distributed about the centre, but this data is left skewed and not bell-shaped (Mean ≠ Median ≠ Mode so ND not valid as peak (mode) 290 – 300 not in centre). e.g. P(X < 250) = 0.016 (ND) but much higher $\frac{7}{70}=0.1$ in data showing larger left tail than ND so not symmetrical. S2: Student could calculate any probability and compare to show skew of data. e.g. 99% of the ND model would be between 243 and 316 km, while the data clearly extends further, especially to the left. S3: This data is not clearly uni-modal, where the normal distribution model would have one central peak. **Evaluation** 1. Clear decision as to whether the claim can be justified or not. For example, "The means seem close, but the sd does not match the claim, so I do not think it is a fair claim." 2. However, the test data was only collected in urban areas. The manufacturer may have used data from a whole range of driving conditions so, even though Figure 1 is not very normal, it is possible that the manufacturer's claim is correct.	TWO valid comments about <u>different</u> aspects of shape, centre, spread or comment on the quality of the testing.	TWO valid **comparative** comments about <u>different</u> aspects of shape, centre, spread with <u>justification</u>.	TWO valid **comparative** comments about <u>different</u> aspects of shape, centre, spread with <u>justification</u>. AND Clear and explicit evaluation of the manufacturer's claim.

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

<table>
<tr><th>Q
THREE</th><th>Evidence</th><th>Achievement
(u)</th><th>Achievement
with Merit (r)</th><th>Achievement
with Excellence
(t)</th></tr>
<tr><td>(a)(i)</td><td>P(EV or PHEV) = $\frac{420}{2000} = 0.21$</td><td>Correct proportion.</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>P(solar among EV owners) = $\frac{104}{275} = 0.3782$
P(solar among PHEV owners) = $\frac{45}{145} = 0.3103$
P(solar among non-electric owners) = $\frac{205}{1580} = 0.1297$
EV owners are more likely to have a home solar system than PHEV or non-electric car owners.</td><td>One correct P(solar) probability found</td><td>All three probabilities correct with conclusion.</td><td></td></tr>
<tr><td>(iii)
&
(iv)</td><td>$\frac{\text{22144 home solar systems}}{\text{Total}}$
P(EV among solar owners) in America= $\frac{104}{354}$
= 0.2938
If these results are valid in NZ we would expect about 29% of the home solar system owners to have EVs, which is 6506 people who would have EVs.
Accept any whole number between 6422 (29%) and 6510 (29.4%)

Possible reasons why this estimate may not be valid:
• Sampling method (online survey of those interested / owners)
• Transference of findings from America to NZ may not be valid as… (differences in technology, pricing, availability etc.)
Sample or population size disparities, or differences in the time that surveys ran for, are not valid reasons.</td><td>Correct probability EV / Solar found
OR
gives at least one valid reason why the estimate may not be valid.</td><td>Correct expected value rounded to whole number
AND
at least one valid reason why the estimate may not be appropriate.</td><td></td></tr>
<tr><td>(b)(i)</td><td>Table 2: Europe
<table><tr><th></th><th>Home solar system</th><th>No solar system</th><th>Total</th></tr><tr><td>Electric Vehicle (EV)</td><td>63</td><td>162</td><td>225</td></tr><tr><td>Plug in Hybrid (PHEV)</td><td>23</td><td>73</td><td>96</td></tr><tr><td>Non-Electric vehicle</td><td>185</td><td>694</td><td>879</td></tr><tr><td>Total</td><td>271</td><td>929</td><td>1200</td></tr></table>P(home solar system among non-electric owners)
= $\frac{185}{879} = 0.2105$</td><td>Probability correct.</td><td></td><td></td></tr>
</table>

(ii)	In America: P(solar among EV owners) $= \frac{104}{275} = 0.3782$ P(solar among non-EV owners) $= \frac{205}{1580} = 0.1297$ Relative Risk $= \frac{0.3782}{0.1297} = 2.91$ times as likely for EV owners to have solar than non-EV owners in America. This is close but slightly under 3 times as likely in the claim. If PHEV included in EV: $\frac{149/420}{205/1580} = \frac{0.3547}{0.1297} = 2.74$ times as likely In Europe: P(solar among EV owners) $= \frac{63}{225} = 0.28$ P(solar among non-EV owners) $= \frac{185}{879} = 0.2105$ Relative Risk $= \frac{0.28}{0.2105} = 1.33$ which means that EV owners in Europe are 33% more likely to have a home solar system than non-EV owners. This is close to the claim of 30%. If PHEV included in EV: $\frac{86/321}{185/879} = \frac{0.2679}{0.2105} = 1.27$ which is 27% more likely. Given that this was only one sample of reasonable size, these relative risks are close enough to the claims to suggest they could be substantiated OR clear discussion of why they are not valid, such as citing that 2.91 is less than 3 for America, or qualifying their claims by citing the fact that online surveys might not be representative of the whole population because of participation bias.	One European probability correct.	One relative risk obtained correctly.	T1 Calculates both relative risks correctly and interprets them and makes a decision on the validity of the claims. T2: Both relative risks calculated and interpreted AND validity of the claims is justified (either way) with at least one valid connection to the context of these surveys.

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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 14	15 – 19	20 – 24