Assessment Schedule – 2019

Mathematics and Statistics: Apply calculus methods in solving problems (91262)

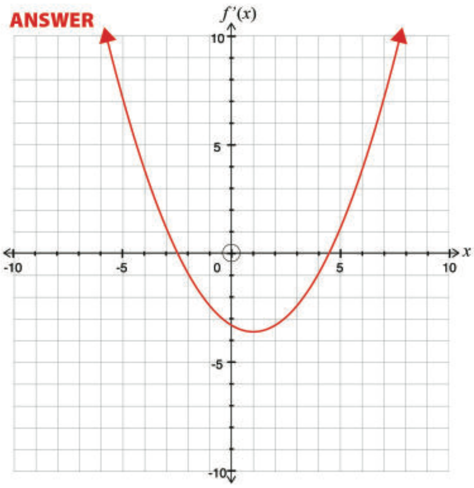
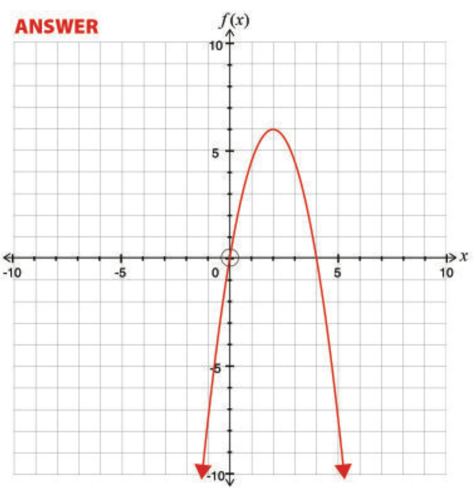
Evidence

ONE	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$f'(x) = 4x^3 + 6x$ $f'(2) = 4 \times 8 + 12$ $= 44$	Derivative found and gradient evaluated.		
(b)	$\frac{dy}{dx} = 12x^2 - 4$ Slope of $y - 8x + 6 = 0$ is 8 when $12x^2 - 4 = 8$ $12x^2 = 12$ so $x^2 = 1$ and $x = \pm 1$ $x = 1, y = 4 - 4 + 4 = 4$ $x = -1, y = -4 + 4 + 4 = 4$ so at (1,4) and (– 1,4)	Derivative found and made equal to 8.	Only one co-ordinate point found.	Both points found.
(c)	$\frac{dV}{dr} = \frac{4}{3} \times 3\pi r^2$ $= 4\pi r^2$ $4\pi r^2 = 25\pi$ $r^2 = 6.25$ $r = 2.5$ cm	Derivative found and made equal to 25π.	Radius found.	
(d)	Let $y = ax^2 + bx + c$ $\frac{dy}{dx} = 2ax + b$ as $\frac{dy}{dx} = 7$ when $x = 0$ $b = 7$ when $x = 1$ $\frac{dy}{dx} = 0$ $2a + 7 = 0$ so $a = -3.5$ When $x = 4$ $-3.5 \times 16 + 7 \times 4 + c = -20$ giving $c = 8$ and $y = -3.5x^2 + 7x + 8$	General form of quadratic, with derivative found.	General form of quadratic, with derivative found and made equal to 7 when $x = 0$, and made equal to 0 when $x = 1$.	Quadratic found correctly.

(e)	At the point of intersection $g(-1) = h(-1)$ and also $g'(-1) = h'(-1)$ $g'(x) = 3x^2 - 2ax$ $h'(x) = 4x + b$ so $3 + 2a = -4 + b$ or $2a - b = -7$ also $-1 - a + 6 = 2 - b + 13$ or $-a + b = 15 + 1 - 6$ $-a + b = 10$ $a = 3$ and $b = 13$ Point of intersection is $(-1, 2)$	Finds either $g'(x)$ AND $g'(-1)$ OR $h'(x)$ AND $h'(-1)$	Derivatives of both g and h found, recognising that they must be equal when $x = -1$ AND recognising that $g(-1) = h(-1)$.	Simultaneous equations are set up to find a and b AND co-ordinates of point of contact found.

N1	N2	A3	A4	M5	M6	E7	E8
Attempt at one question.	1 of u.	2 of u.	3 of u.	1 of r.	2 of r.	1 of t.	2 of t.

N0 = No response; no relevant evidence.

TWO	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)		Upright parabola with x intercepts lined up with turning points of original graph between – 3 and – 2 and 4 and 5 OR turning point midway between these or lined up with steepest gradient of original ($x = 1$).	Upright parabola with x intercepts lined up with turning points of original graph between – 3 and – 2 and 4 and 5 AND turning point midway between these or lined up with steepest gradient of original ($x = 1$).	
(b)	$p(x) = 5x - \frac{8x^4}{4} + c$ $p(x) = 5x - 2x^4 + c$ $-25 = 10 - 32 + c$ and $p(x) = 5x - 2x^4 - 3$	Correct polynomial in any form.		
(c)	$f'(x) = -6k^2x^2 + 6kx + 12$ $f'(x) > 0$ means $-6(k^2x^2 - kx - 2) > 0$ $(kx + 1)(kx - 2) < 0$ Function is increasing for $\frac{-1}{k} < x < \frac{2}{k}$ Justification: Shape of original cubic OR shape of parabolic derivative OR check gradient at a point.	Derivative found and made greater than 0.	Correct endpoints of interval.	Correct range and justification for the choice of that range of values for k.
(d)		Inverted parabola either going through (0,0) and (4,0) OR with turning point lined up with x-intercept.	Inverted parabola going through (0,0) and (4,0) AND with turning point lined up with x-intercept.	

(e)	$a = -4$ $v = -4t + c$ When $t = 0$, $v = 28$ So $v = -4t + 28$ $s = 28t - \frac{4t^2}{2} + c$ $= 28t - 2t^2 + c$ $28 - 4t = 10$ means $t = 4.5$, so it takes 4.5 seconds for the car to reach the corner. When $t = 4.5$, $s = 0$ Here s is the distance from the corner. So $0 = 28 \times 4.5 - 2 \times (4.5)^2 + c$ $c = -85.5$ so $s = 28t - \frac{4t^2}{2} - 85.5$ i.e. the car was 85.5 metres away from the corner when the brakes were first applied.	Finds expression for velocity by anti-differentiating.	Finds general expression for s and solves to find the time it takes the car to reach the corner.	Finds value of c and correctly finds distance car was from corner.

N1	**N2**	**A3**	**A4**	**M5**	**M6**	**E7**	**E8**
Attempt at one question.	1 of u.	2 of u.	3 of u.	1 of r.	2 of r.	1 of t.	2 of t.

N0 = No response; no relevant evidence.

THREE	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	$v = 6 - 2t$, so initial speed is 6 m s^{-1}	Derivative found and speed found when $t = 0$.		
(a)(ii)	Initially speed is positive. $6 - 2t = 0$ when $t = 3$ So speed is zero when $t = 3$ seconds. After 3 seconds its speed is negative as $6 - 2t < 0$ when $t > 3$ This means that the car is heading back to the start after three seconds.	Finds the value of t OR $v = 0$ is identified.	Change of direction justified: either velocity found before and after $t = 3$, or maximum given when $v = 0$ at turning point.	
(a)(iii)	It reaches P when $s = 0$ $t(6 - t) = 0$ means $t = 0$ or $t = 6$. When $t = 6$, $v = 6 - 12 = -6$ m s^{-1}	Finds $t = 6$ when it reaches P for the second time and finds the speed.	Interprets that the speed is 6 m s^{-1} in the opposite direction.	
(b)	Let l be length of rectangular side, and x be length of one of the square sides. $l + 4x = 100$ $l = 100 - 4x$ $V = lx^2$ $= (100 - 4x)x^2$ $= 100x^2 - 4x^3$ $\frac{dV}{dx} = 200x - 12x^2$ $= x(200 - 12x)$ $x = 0$ means $V = 0$, so for max volume, $12x = 200$ $x = 16.666$ cm Maximum possible volume is 9259 cm^3. Accept any clear justification of the fact that this is the maximal volume. Methods include noting that the other turning point leads to zero volume, second derivative test, testing gradients in the neighbourhood, referring to shape of curve.	Sets up equation for volume in terms of one variable, and differentiates.	Makes derivative equal 0, solves the quadratic derivative with two solutions, for x.	Finds correct maximum volume with justification.

(c)	$\frac{dy}{dx} = 3x^2 - 6x - 4$ When $x = a$, the gradient of the tangent is $3a^2 - 6a - 4$ If the tangent when $x = a$ goes through the origin, then $f(a)/a$ = gradient of tangent. $\frac{a^3 - 3a^2 - 4a}{a} = (3a^2 - 6a - 4)$ $a^3 - 3a^2 - 4a = 3a^3 - 6a^2 - 4a$ $2a^3 - 3a^2 = 0$ $a^2(2a - 3) = 0$ $a = 0$ or $a = \frac{3}{2}$ Points are (0, 0) and (1.5, – 9.375) with tangents $y = -4x$ and $y = -6.25x$ respectively.	Finds derivative of f and makes some valid progress.	Finds one point and the equation of the tangent.	Finds both points and equations of tangents.

N1	N2	A3	A4	M5	M6	E7	E8
Attempt at one question.	1 of u.	2 of u.	3 of u.	1 of r.	2 of r.	1 of t.	2 of t.

N0 = No response; no relevant evidence.

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 19	20 – 24