Assessment Schedule – 2019

Mathematics and Statistics: Apply algebraic methods in solving problems (91261)

Assessment Criteria

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems involves: • selecting and using methods • demonstrating knowledge of algebraic concepts and terms • communicating using appropriate representations.	*Apply algebraic methods, using relational thinking, in solving problems* must involve one or more of: • selecting and carrying out a logical sequence of steps • connecting different concepts or representations • demonstrating understanding of concepts • forming and using a model and also relating findings to a context, or communicating thinking using appropriate mathematical statements.	*Apply algebraic methods using extended abstract thinking, in solving problems* involves one or more of: • devising a strategy to investigate or solve a problem • identifying relevant concepts in context • developing a chain of logical reasoning, or proof • forming a generalisation and also using correct mathematical statements or communicating mathematical insight.

Evidence

ONE	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	$3x^2 - 7x - 6 = (3x + 2)(x - 3) = 0$ $x = \frac{-2}{3}$, 3 or equivalent.	Correct solutions.		
(a)(ii)	$5x^2 - 4x - 3 = 0$ $x = 1.27, -0.47$ or equivalent.	Correct solutions.		
(b)	$5.05 = 0.02t^2 - 0.6t + 9.18$ $0.02t^2 - 0.6t + 4.13 = 0$ $t = 10.7, 19.3$ Hence $t = 10.7$ months (cannot be 19.3)	Quadratic equation set equal to 0.	Correct answer.	
(c)	Discriminant $\Delta = b^2 - 4ac = 0$ $(m + 1)^2 - 4(2m - 1)(m - 4) = 0$ $7m^2 - 38m + 15 = 0$ $(7m - 3)(m - 5) = 0$ $m = 5$ ($\frac{3}{7}$ eliminated as $m - 4$ must be at least 0 since the expression is a perfect square)	Sets up discriminant equal to 0.	Sets up quadratic equation equal to 0.	Correct solution incorporating clear explanation of the rejection of $m = \frac{3}{7}$.

(d)	$p^2x^2 + 4px - 12 = (px + 6)(px - 2) = 0$ $x = \frac{-6}{p}, \frac{2}{p}$ Hence $\frac{2}{p} - \frac{-6}{p} = \frac{8}{p}$.	Factorises correctly or finds the difference between roots consistently.	The correct roots are found.	Finds the difference between the correct roots.
(e)	$y = x^2 - bx - ax + ab - c^2$ $= x^2 - (a + b)x + (ab - c^2)$ $\Delta = (-(a + b))^2 - 4 \times 1 \times (ab - c^2)$ $= (a^2 - 2ab + b^2) + 4c^2$ $= (a - b)^2 + 4c^2$ As $(a - b)^2 \geq 0$ and $c^2 > 0$ then $\Delta > 0$ and hence there are two real distinct roots (and two distinct points where the graph crosses the x–axis). Graphical argument: "Since $f(x) = (x - a)(x - b)$ is a positive parabola which clearly has 2 roots and its graph crosses the x-axis at 2 distinct points, the function $g(x) = f(x) - c^2$, which must be lower since c^2 must be positive, must also have 2 distinct roots (which would be further apart than a and b)."	Function is set up so that the discriminant can be found.	Finds discriminant in factored form.	Explains why the discriminant is greater than 0 and makes a correct conclusion. OR graphical argument is fully described.

N1	N2	A3	A4	M5	M6	E7	E8
A valid attempt at one question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

N0 = No response; no relevant evidence.

TWO	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	$9^{0.5}a^1b^{-2}$ $= \frac{3a}{b^2}$	Correct answer.		
(a)(ii)	$\left(\frac{3b^4}{2a}\right)^2 = \frac{9b^8}{4a^2}$	Correct answer.		
(b)	$\frac{2c+1}{(c+3)(c-3)} + \frac{c-1}{(c-3)(c-1)}$ $= \frac{(2c+1)(c-1)+(c-2)(c+3)}{(c+3)(c-3)(c-1)}$ $= \frac{3c^2-7}{(c+3)(c-3)(c-1)}$	Cross-arrangement to a single fraction.	Final simplification.	
(c)	$fm - 2gm - 6gn + 3fn$ $= m(f-2g) + 3n(f-2g)$ $= (m+3n)(f-2g)$	Pairs factored.	Complete factorisation.	
(d)	Small rectangle: Area = $y^2 - 8y = 9$ $y^2 - 8y - 9 = (y-9)(y+1) = 0 \Rightarrow y = 9$ Since $x = 2y - 6$ Large rectangle: Area = $(2y-6)(2y-10)$ $= 4y^2 - 32y + 60$ Hence Area = 96 cm^2	y found.		Area found.
(e)	Roots are $\frac{-p+\sqrt{p^2-4q}}{2}$ and $\frac{-p-\sqrt{p^2-4q}}{2}$ $-p+\sqrt{p^2-4q} = n(-p-\sqrt{p^2-4q})$ $\sqrt{p^2-4q} = \frac{p(1-n)}{(1+n)}$ $qn^2+(2q-p^2)n+q=0$	One root n times the other.	Successfully squares both sides.	Finds equation with correct algebraic working.

N1	N2	A3	A4	M5	M6	E7	E8
A valid attempt at one question	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

N0 = No response; no relevant evidence.

THREE	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\log_5(m) = 3$ $\Leftrightarrow\ 5^3 = m \Leftrightarrow m = 125$	Correct answer.		
(b)	$\log 6 - 2\log y$ $= \log\left(\dfrac{6}{y^2}\right)$	Correct answer.		
(c)	$\dfrac{3^{2n-1}+3^{2n+1}}{3^{2n}-3^{2n-4}} = \dfrac{3^{2n-4}(3^3+3^5)}{3^{2n-4}(3^4-1)}$ $= \dfrac{27+243}{81-1} = \dfrac{27}{8}$ or equivalent.	Correct answer only.	Finds common factor of numerator and denominator.	Correct answer.
(d)(i)	$3N_0 = N_0(1.053)^t$ $3 = (1.053)^t$ $\log(3) = t\log(1.053)$ $t = \dfrac{\log(3)}{\log(1.053)} = 21.27$ weeks	Taking log of both sides and t as a factor.	Correct answer.	
(d)(ii)	$\dfrac{4250}{2500} = (1+\dfrac{r}{100})^{10}$ $1+\dfrac{r}{100} = \sqrt[10]{1.7} = 1.0545$ Hence r = 5.45 and rate of change is 5.45%.	Sets up correct equation.	Finds $1+\dfrac{r}{100}$.	Percentage rate of change found.
(e)	$k = \dfrac{227-67}{(640)^2} = 0.000390625$ $h = 0.000390625\,(x-640)^2 + 67 = 100$ $0.000390625x^2 - 0.5x + 127 = 0$ $x = 349.3,\ 930.7$ m Required distance = 349.3 m		Finds k OR finds two consistent solutions from calculated k.	Correct answer.

N1	N2	A3	A4	M5	M6	E7	E8
A valid attempt at one question	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

N0 = No response; no relevant evidence.

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 14	15 – 19	20 – 24