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Level 3 Calculus 2020

91578 Apply differentiation methods in solving problems

9.30 a.m. Monday 23 November 2020
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

(a) Differentiate $y = \left(3x - x^2\right)^5$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y = 3\sin 2x + \cos 2x$ at the point where $x = \dfrac{\pi}{4}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Find the value of x for which the graph of the function $y = \dfrac{x}{1 + \ln x}$ has a stationary point.

You must use calculus and show any derivatives that you need to find when solving this problem.

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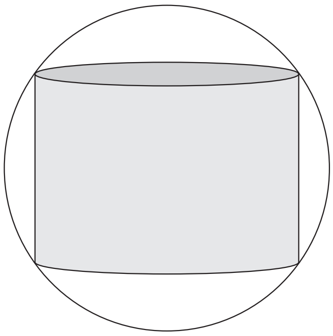
(d) A curve has the equation $y = x^2 \cos x$.

Show that the equation of the tangent to the curve at the point $\left(\pi, -\pi^2\right)$ is

$$y + 2\pi x = \pi^2$$

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) A cylinder of height h and radius r is inscribed, as shown to the right, inside a sphere of radius 20 cm.



Find the maximum possible volume of the cylinder.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the volume you have found is a maximum.

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QUESTION TWO

(a) Differentiate $y = \dfrac{\tan x}{x^3}$.

You do not need to simplify your answer.

(b) The value of a car is modelled by the formula

$$V = 17\,000\,\mathrm{e}^{-0.25t} + 2\,000\,\mathrm{e}^{-0.5t} + 500 \quad \text{for } 0 \le t \le 20$$

where V is the value of the car in dollars ($), and t is the age of the car in years.

Calculate the rate at which the value of the car is changing when it is 8 years old.

You must use calculus and show any derivatives that you need to find when solving this problem.

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(c) Find the x-coordinates of any stationary points on the graph of the function

$$f(x) = (2x - 3)e^{x^2 + k}$$

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) A rocket is fired vertically upwards. Its height above the launch point is given by the formula $h(t) = 4.8t^2$, where h is the height in metres, and t is the time in seconds from firing.

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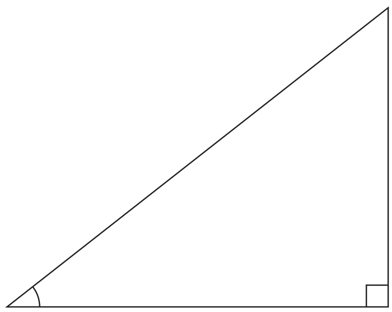

An observer at point A is watching the rocket. She is at the same level as the launch point of the rocket, and 500 m from the launch point.

Find the rate at which the angle of elevation at A of the rocket is increasing when the rocket is 480 m above the launch point.

You must use calculus and show any derivatives that you need to find when solving this problem.

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(e) A curve is defined by the parametric equations $x = \ln(t)$ and $y = 6t^3$ where $t > 0$.

The point P lies on the curve, and at point P, $\frac{d^2y}{dx^2} = 2$.

Find the exact coordinates of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION THREE

(a) Differentiate $y = 3\ln\left(x^2 - 1\right)$.

You do not need to simplify your answer.

(b) For what value(s) of *x* does the tangent to the graph of the function $f(x) = 2x - 2\sqrt{x},\ x > 0$, have a gradient of 1?

You must use calculus and show any derivatives that you need to find when solving this problem.

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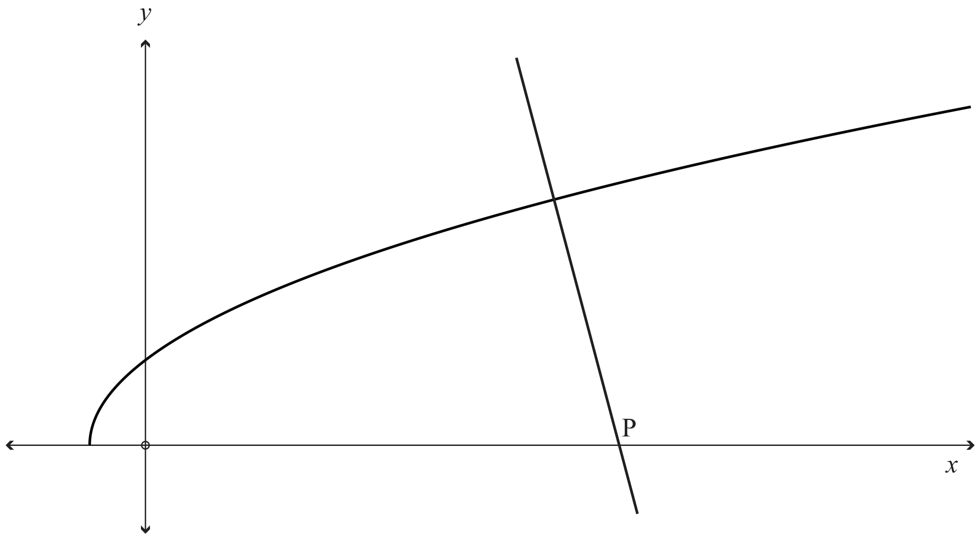
(c) The normal to the graph of the function $y = \sqrt{2x+1}$ at the point $(4,3)$ intersects the x-axis at point P.



Find the x-coordinate of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the following page.

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(d) The graph of the function $y = \frac{1}{x-3} + x,\ x \neq 3$, has two stationary points.

Find the x-coordinates of the stationary points, and determine whether they are local maxima or local minima.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) A curve has the equation $y = (3x + 2)e^{-2x}$.

Prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

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Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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