3

SUPERVISOR'S USE ONLY

91578

915780

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Calculus, 2019

91578 Apply differentiation methods in solving problems

9.30 a.m. Tuesday 26 November 2019
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Differentiate $y = \sqrt{3x^2 - 1}$.

You do not need to simplify your answer.

(b) Find the rate of change of the function $f(t) = 5\ln(3t - 1)$ when $t = 4$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) Find the gradient of the tangent to the curve $y = \dfrac{e^{2x}}{1+x^2}$ at the point where $x = 2$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) For what value(s) of x is the function $y = x^3e^x$ decreasing?

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) The volume of a sphere is increasing.

At the instant when the sphere's radius is 0.5 m, the surface area of the sphere is increasing at a rate of $0.4 \text{ m}^2 \text{ s}^{-1}$.

Find the rate at which the volume of the sphere is increasing at this instant.

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Differentiate $y = (2x - 5)^4$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y = \tan 2x$ at the point on the curve where $x = \frac{\pi}{6}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A curve is defined parametrically by the equations $x = \frac{1}{(5-t)^2}$ and $y = 5t - t^2$.

Find the gradient of the tangent to the curve at the point when $t = 2$.

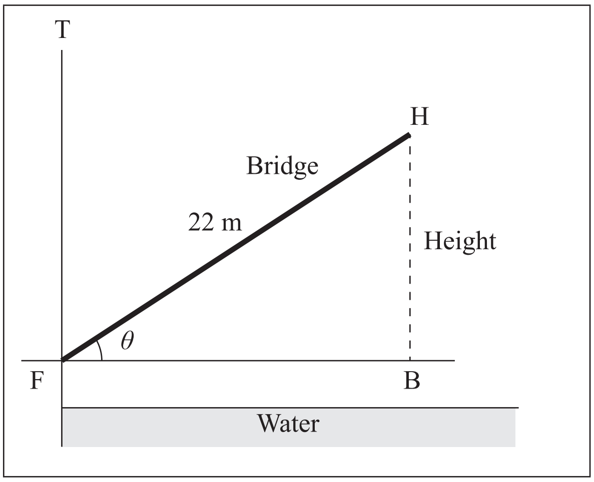
You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

(d) The Wynyard Crossing bridge in Auckland can be raised and lowered to allow tall boats to sail through when open, and pedestrians to walk across when closed. The bridge consists of two arms, each of length 22 metres.

When the bridge is rising, the angle of the bridge arm above the horizontal increases at the rate of 0.01 rad s^{-1}.

www.youtube.com/watch?v=Q4xrCt-uYPE



Find the rate at which the height, BH, is increasing when H is 15 metres above the horizontal, FB.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) If $y = e^u$ and $u = \sin 2x$ show that

$$\frac{d^2y}{dx^2} = \frac{d^2y}{du^2}\left(\frac{du}{dx}\right)^2 + \frac{dy}{du}\frac{d^2u}{dx^2}$$

You must use calculus and show any derivatives that you need to find when solving this problem.

ASSESSOR'S USE ONLY

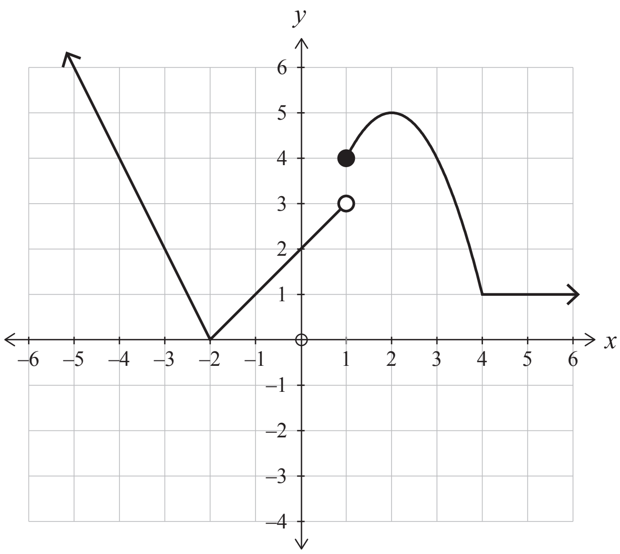
ASSESSOR'S USE ONLY

QUESTION THREE

(a) Differentiate $y = \dfrac{4}{\sin x}$.

You do not need to simplify your answer.

(b) The graph below shows the function $y = f(x)$.



(i) Find all the value(s) of x which meet each of the following conditions:

1. $f'(x) = 0$:

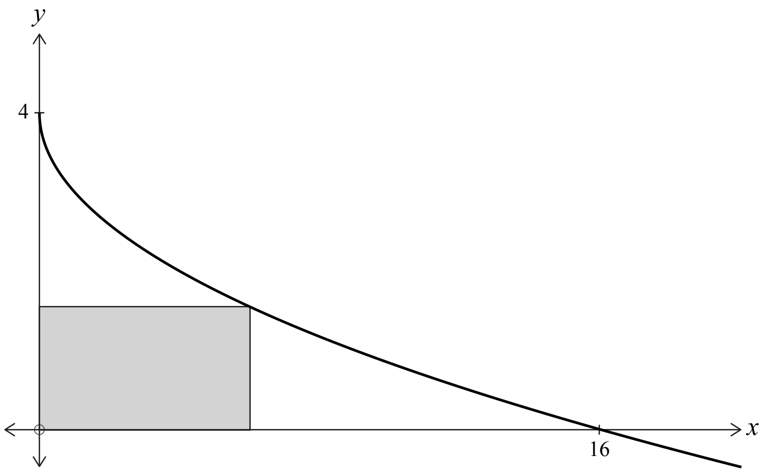
2. $f(x)$ is not differentiable:

(ii) What is the value of $\lim\limits_{x \to 1} f(x)$?

State clearly if the value does not exist.

ASSESSOR'S USE ONLY

(c) A rectangle has one vertex at (0,0), and the opposite vertex on the curve $y = 4 - \sqrt{x}$, where $0 < x < 16$, as shown on the graph below.



Find the maximum possible area of the rectangle.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the area you have found is a maximum.

ASSESSOR'S USE ONLY

(d) The velocity of an object is modelled by the function

$$v = 2e^{t} + 8e^{-t}, \text{ for } t \geq 0$$

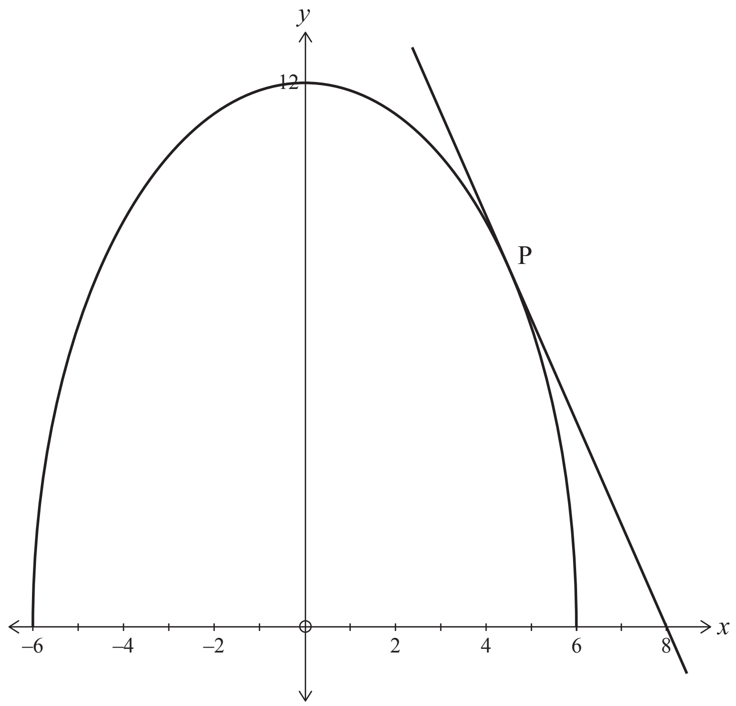
where v is the velocity of the object, in m s^{-1}
and t is the time in seconds since the start of the object's motion.

Find the time when the acceleration of the object is 0.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the following page.

(e) The graph below shows the function $y = 2\sqrt{36 - x^2}$, and the tangent to that function at point P. The tangent intersects the x-axis at the point (8,0).



Find the x-coordinate of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

91578