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91578

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 3 Calculus, 2014

91578 Apply differentiation methods in solving problems

9.30 am Tuesday 18 November 2014
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) Differentiate $y = 5\cos(3x)$.

(b) Find the gradient of the normal to the function $y = (3x^2 - 5x)^2$ at the point (1,4).

Show any derivatives that you need to find when solving this problem.

(c) If $x = 2\sin t$ and $y = \cos 2t$ show that $\frac{dy}{dx} = -2\sin t$.

(d) Find the x-value at which the tangent to the function $y = \frac{4}{e^{2x-2}} + 8x$ is parallel to the x-axis.

Show any derivatives that you need to find when solving this problem.

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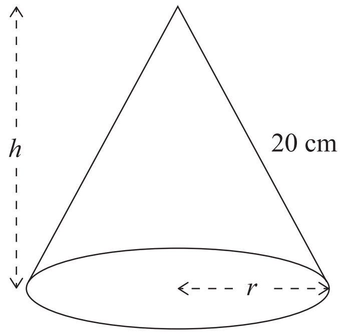
(e) What is the maximum volume of a cone if the slant length of the cone is 20 cm?



You do not need to prove that the volume you have found is a maximum.

Show any derivatives that you need to find when solving this problem.

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QUESTION TWO

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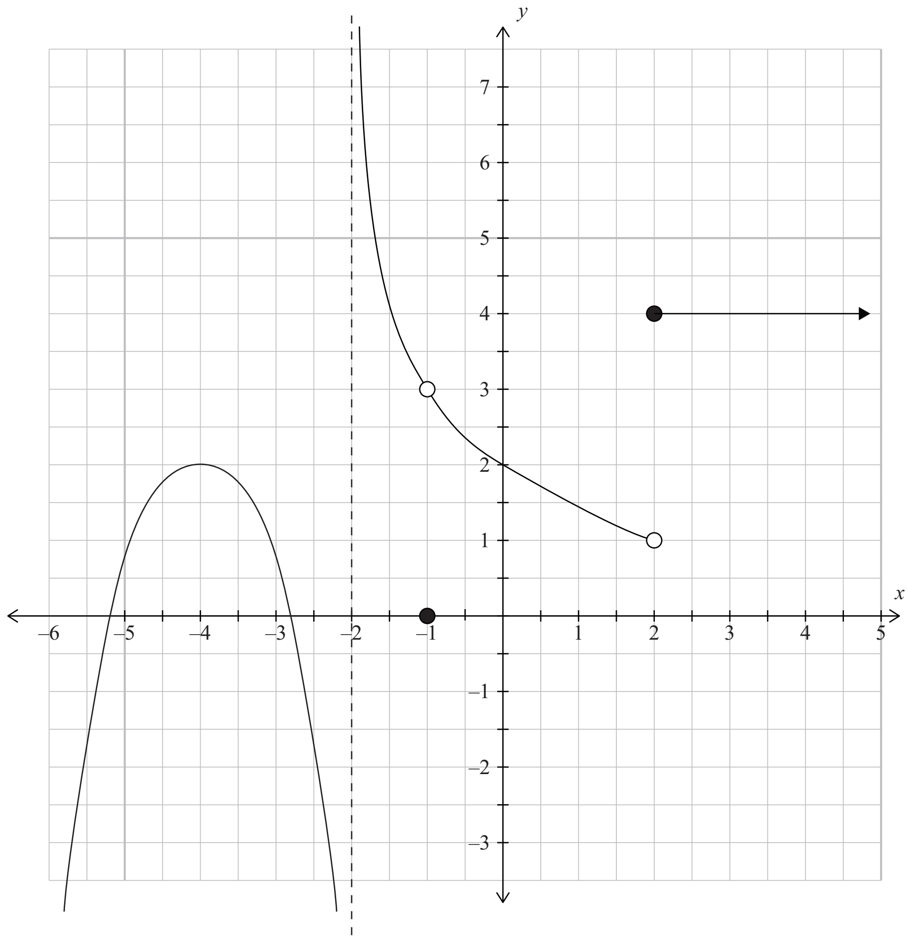
(a) Differentiate $f(x) = \dfrac{e^{4x}}{2x-1}$.

You do not need to simplify your answer.

(b) Find the gradient of the curve defined by $y = 8\ln(3x - 2)$ at the point where $x = 2$.

Show any derivatives that you need to find when solving this problem.

(c) The graph below shows the function $y = f(x)$.



For the function $f(x)$ above:

(i) Find the value(s) for x that meet the following conditions:

1. $f(x)$ is not differentiable: ______

2. $f''(x) < 0$: ______

3. $f(x)$ is not defined: ______

(ii) What is the value of $f(2)$? ______

State clearly if the value does not exist.

(iii) What is the value of $\lim_{x \to -1} f(x)$? ______

State clearly if the value does not exist.

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(d) The hourly cost of running an aeroplane depends on the speed at which it flies.

For a particular aeroplane this is given by the equation

$$C = 4v + \frac{1\,000\,000}{v}, \;\; 200 \le v \le 800$$

where C is the hourly cost of running the aeroplane, in dollars per hour and v is the airspeed of the aeroplane, in kilometres per hour.

Find the minimum hourly cost at which this aeroplane can be flown.

Show any derivatives that you need to find when solving this problem.

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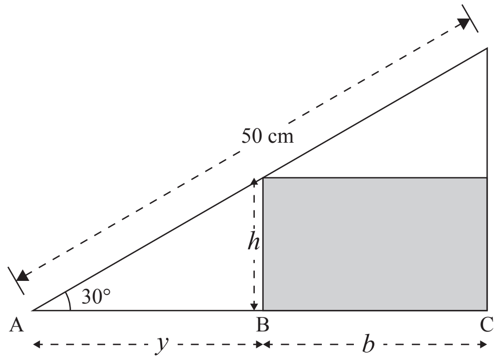
(e) A rectangle is drawn inside a right angled triangle, as shown in the diagram below.



Point B moves along the base of the triangle AC, beginning at point A, at a constant speed of 3 cm s^{-1}.

At what rate is the area of the rectangle changing when point B is 20 cm from point A?

Show any derivatives that you need to find when solving this problem.

QUESTION THREE

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(a) Differentiate $y = \left(\sqrt[3]{x^2 + 4x}\right)^2$.

(b) Find the value(s) of x for which the graph of the function $y = x + \frac{32}{x^2}$ has stationary points.

Show any derivatives that you need to find when solving this problem.

(c) For what values of x is the function $f(x) = 5x - x\ln x$ increasing?

Show any derivatives that you need to find when solving this problem.

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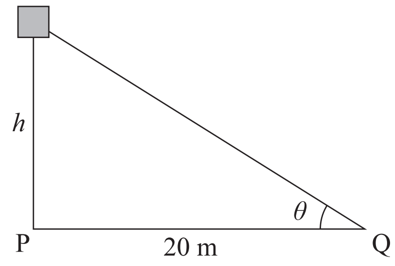
(d) A container is winched up vertically from a point P at a constant rate of 1.5 m s^{-1}.

It is being observed from point Q, which is 20 m horizontally from point P.

θ is the angle of elevation of the container from point Q.

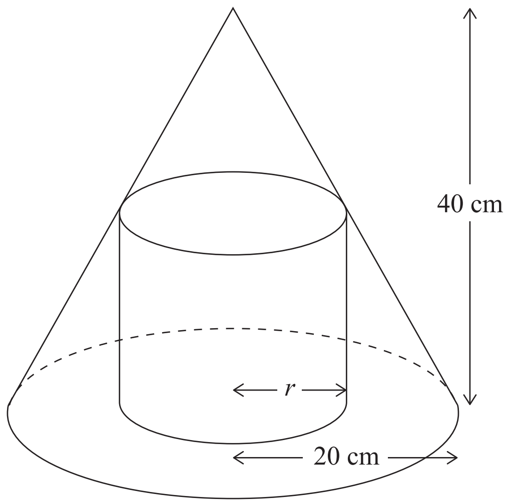


At what rate is the angle of elevation increasing when the object is 20 m above point P?

Show any derivatives that you need to find when solving this problem.

(e) A cone has a radius of 20 cm and a height of 40 cm.

A cylinder fits inside the cone, as shown below.



What must the radius of the cylinder be to give the cylinder the maximum volume?

You do not need to prove that the volume you have found is a maximum.

Show any derivatives that you need to find when solving this problem.

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Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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