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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

SUPERVISOR'S USE ONLY

Level 3 Calculus, 2013

91578 Apply differentiation methods in solving problems

9.30 am Wednesday 13 November 2013
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

You are advised to spend 60 minutes answering the questions in this booklet.

ASSESSOR'S USE ONLY

QUESTION ONE

(a) Differentiate $y = \tan(x^2 + 1)$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the function $f(x) = \ln(3x - e^x)$ at the point where $x = 0$.

(c) Find the x values of any points of inflection on the graph of the function $y = e^{(6 - x^2)}$.

Show any derivatives that you need to find when solving this problem.

(d) A curve is defined by the parametric equations:

$x = 5\sin t$ and $y = 3\tan t$

Find the gradient of the normal to the curve at the point where $t = \frac{\pi}{3}$.

Show any derivatives that you need to find when solving this problem.

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(e) A closed cylindrical tank is to have a surface area of 20 m^2.

Find the radius the tank needs to have so that the volume it can hold is as large as possible.

You do not have to prove that your solution gives the maximum volume.

Show any derivatives that you need to find when solving this problem.

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QUESTION TWO

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(a) Differentiate $y = \sqrt[3]{\pi - x^2}$.

You do not need to simplify your answer.

(b) A curve has the equation y = $(x^3 - 2x)^3$.

Find the equation of the tangent to the curve at the point where $x = 1$.

Show any derivatives that you need to find when solving this problem.

(c) For what value of k does the function $f(x) = x - e^x - \frac{k}{x}$ have a stationary point at $x = -1$?

Show any derivatives that you need to find when solving this problem.

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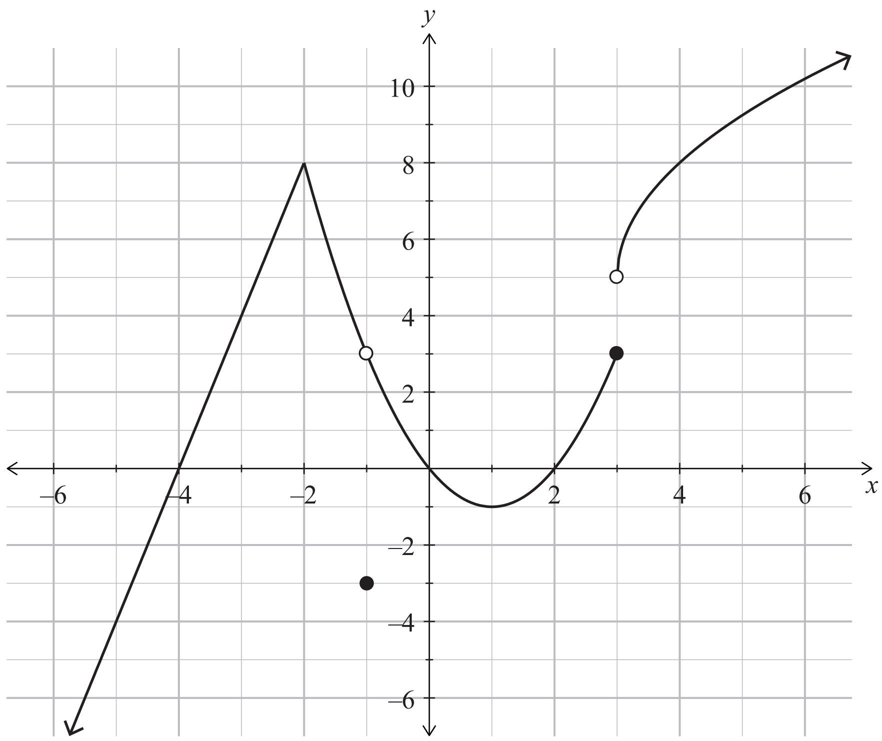
(d) The graph below shows the function $y = f(x)$.



For the function *f(x)* above:

(i) Find all the value(s) of x that meet each of the following conditions:

1. $f'(x) = 0$ ________________

2. $f''(x) < 0$ ________________

3. $f(x)$ is not differentiable ________________

(ii) What is the value of $f(-1)$? ________________

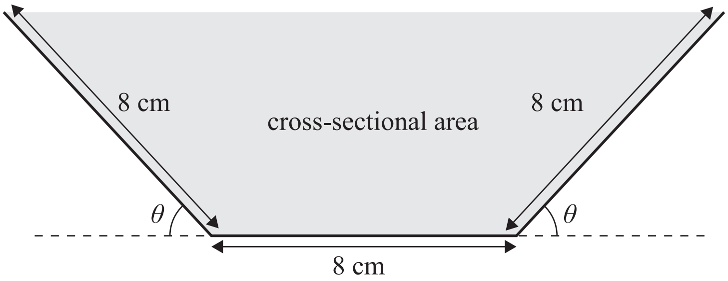
(iii) What is the value of $\lim_{x \to 3} f(x)$?

State clearly if the value does not exist.

____

(e) A copper sheet of width 24 cm is folded, as shown, to make spouting.

Cross-section:



Find angle θ which gives the maximum cross-sectional area.

You do not need to prove that you have found a maximum.

Show any derivatives that you need to find when solving this problem.

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QUESTION THREE

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(a) Differentiate $y = \frac{\sin(2x)}{x^2}$.

You do not need to simplify your answer.

(b) For the function $f(x) = x + \frac{16}{x-2}$, find the x-values of any stationary points.

You must use calculus and clearly show your working, including any derivatives you need to find when solving this problem.

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(c) Find the value of x that gives the maximum value of the function

$$f(x) = 50x - 30x \ln 2x$$

You do not need to prove that your value of x gives a maximum.

You must use calculus and clearly show your working, including any derivatives you need to find when solving this problem.

(d) A curve is defined by the parametric equations:

$x = t^2 - t$ and $y = t^3 - 3t$

Find the coordinates of the point(s) on the curve for which the normal to the curve is parallel to the y-axis.

You must use calculus and clearly show your working, including any derivatives you need to find when solving this problem.

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(e) A spherical balloon is being inflated with helium.

The balloon is being inflated in such a way that its volume is increasing at a constant rate of $300\ \text{cm}^3\ \text{s}^{-1}$.

The material that the balloon is made of is of limited strength, and the balloon will burst when its surface area reaches $7\,500\ \text{cm}^2$.

Find the rate at which the surface area of the balloon is increasing when it reaches bursting point.

Show any derivatives that you need to find when solving this problem.

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Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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