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Level 3 Calculus, 2017

91577 Apply the algebra of complex numbers in solving problems

9.30 a.m. Thursday 23 November 2017
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) If $u = 2 + 3i$ and $v = 1 - 4i$, find $\bar{u} - 3v$, giving your solution in the form $a + bi$.

(b) Write $\dfrac{36}{5-\sqrt{7}}$ in the form $a+b\sqrt{7}$, where a and b are integers.

(c) Solve the following equation for x in terms of p:

$$p\sqrt{x-2} - 5\sqrt{x} = 0$$

(d) One solution of the equation $z^3 - 2z^2 + Bz - 30 = 0$ is $z = -2 - i$.

If B is a real number, find the value of B and the other two solutions of the equation.

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(e) Find the Cartesian equation of the locus described by $|z + 2 - 7i| = 2|z - 10 + 2i|$.

Write your answer in the form $(x + A)^2 + (y + B)^2 = K$.

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QUESTION TWO

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(a) Dividing $x^3 - 2x^2 + 5x + d$ by $(x - 3)$ gives a remainder of 13.

Find the value of d.

(b) Simplify, as far as possible, the expression $\sqrt{2k}\left(\sqrt{18k} - \sqrt{8k}\right)$.

(c) z and w are complex numbers such that $z = -2 + 3\text{i}$ and $zw = 15 - 3\text{i}$.

Find an exact value of $\arg(w)$.

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(d) Solve the equation $z^4 = \frac{m}{\sqrt{2}} + \frac{m}{\sqrt{2}}\text{i}$, where m is real and positive.

Write your solutions in polar form in terms of m.

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(e) Find all possible values of k that make $u = \dfrac{k+4\mathrm{i}}{1+k\mathrm{i}}$ a purely real number.

QUESTION THREE

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(a) If $u = p^3\text{cis}\frac{\pi}{3}$ and $v = p\,\text{cis}\frac{\pi}{8}$, write $\frac{u}{v}$ in polar form.

(b) Solve the equation $x^2 - 6x + 14 = 0$.

Give your solution in the form $a \pm \sqrt{b}\,\text{i}$, where a and b are rational numbers.

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(c) $\dfrac{3x^3+8x^2-2x+11}{x+2} = 3x^2+Ax+B+\dfrac{C}{x+2}$, where A, B, and C are integers.

Find the values of A, B, and C.

(d) Solve the equation $\dfrac{8+x}{x} = \sqrt{3}$, writing your solution in the form $x = a+b\sqrt{3}$.

Question Three continues on the following page.

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(e) z is a complex number such that $z = \dfrac{a + b\mathrm{i}}{a - b\mathrm{i}}$, where a and b are real numbers.

Prove that $\dfrac{z^2 + 1}{2z} = \dfrac{a^2 - b^2}{a^2 + b^2}$.

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Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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