91262

912620

2

SUPERVISOR'S USE ONLY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 2 Mathematics and Statistics, 2013

91262 Apply calculus methods in solving problems

2.00 pm Monday 18 November 2013
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

You are advised to spend 60 minutes answering the questions in this booklet.

ASSESSOR'S USE ONLY

QUESTION ONE

(a) A function f is given by $f(x) = 4x^2 - 5x + 2$.

Find the gradient of the graph of f at the point where $x = 3$.

(b) For a function g,

$g'(x) = 6x^2 - 5$.

The graph of g passes through the point $(1,4)$.

Find the function $g(x)$.

ASSESSOR'S USE ONLY

(c) An emergency flare is fired from a boat.

Its height, h metres above the surface of the water, is given by

$$h = 90t - 5t^2 + 2$$

where t is the time in seconds since the flare was fired.

What is the maximum height reached by the flare?

(d) The distance around a tree (its girth) g metres, at a time t years after it is planted, is modelled by the function

$$g = -0.005t^2 + 0.15t + 0.3 \qquad 0 \leq t \leq 15.$$

When will the rate of growth of the tree's girth be 0.04 metres per year?

ASSESSOR'S USE ONLY

(e) $g(x) = -x^3 + 3x + 2$

For what values of x is g a decreasing function?

You must **show the use of calculus** in your working.

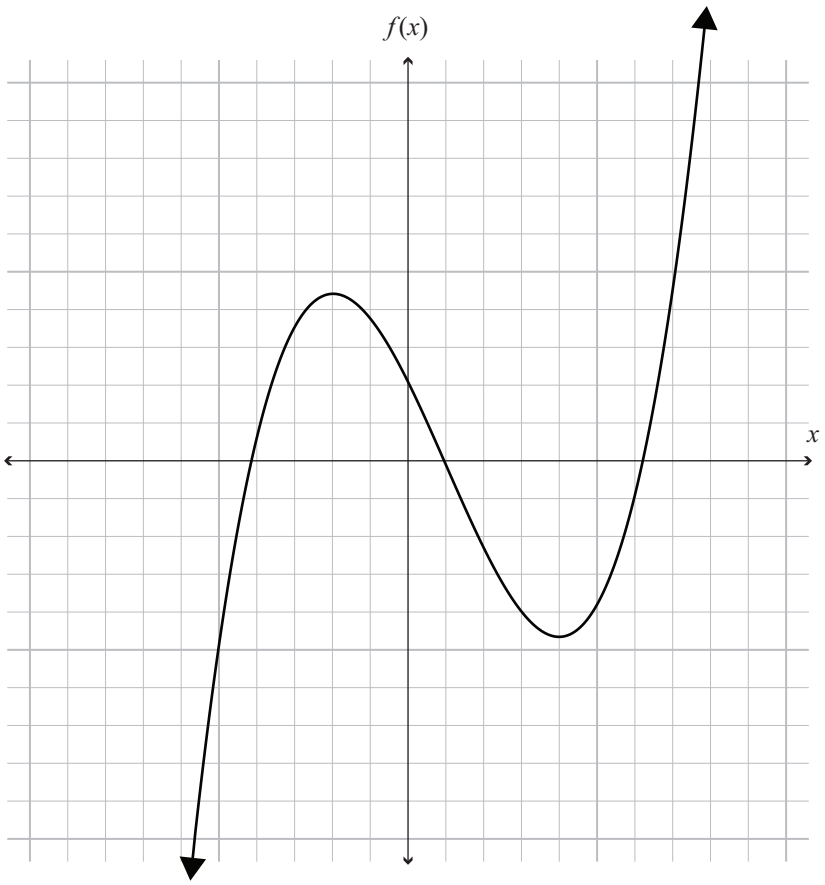
(f) A curve has gradient function $f'(x) = mx + 2$. The curve passes through the points (2,10) and (–1,– 8).

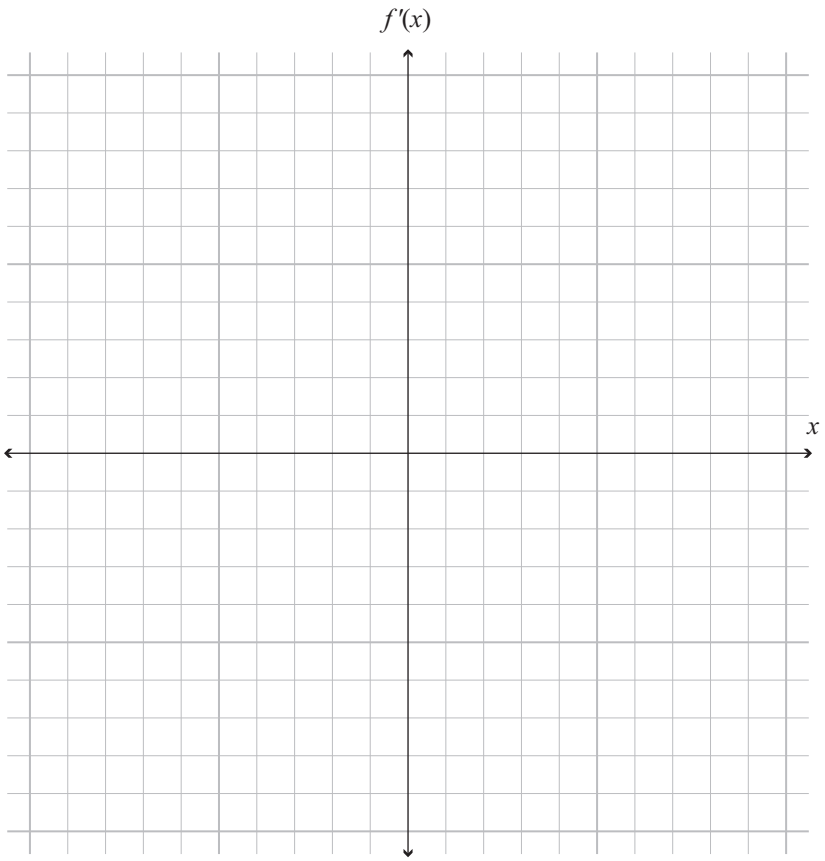
Find $f(x)$, the equation of the curve.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) Sketch the gradient function $f'(x)$ for the function $f(x)$ below:





If you need to redraw this graph, use the grid on page 15

ASSESSOR'S USE ONLY

(b) A tank is being filled with milk. The depth of the milk d cm, at a time t minutes after pouring started is given by

$$\mathrm{d}(t) = \frac{t^2}{4} + t$$

Find the rate at which the depth of the milk is changing 5 minutes after the pouring started.

(c) A stone is dropped into a pool.

This makes circular ripples on the surface of the water.

The area A of a circular ripple, in square metres, is given by

$$A = \pi r^2$$

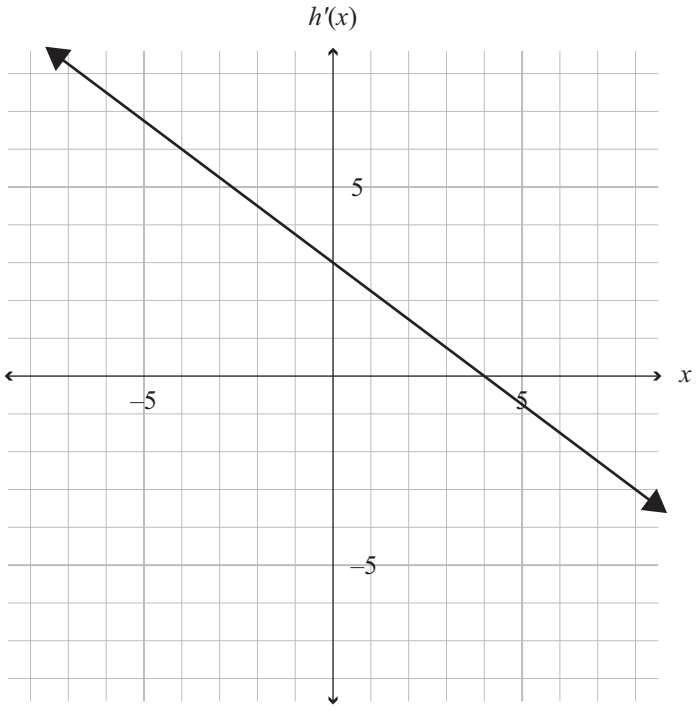
where the radius is r metres.

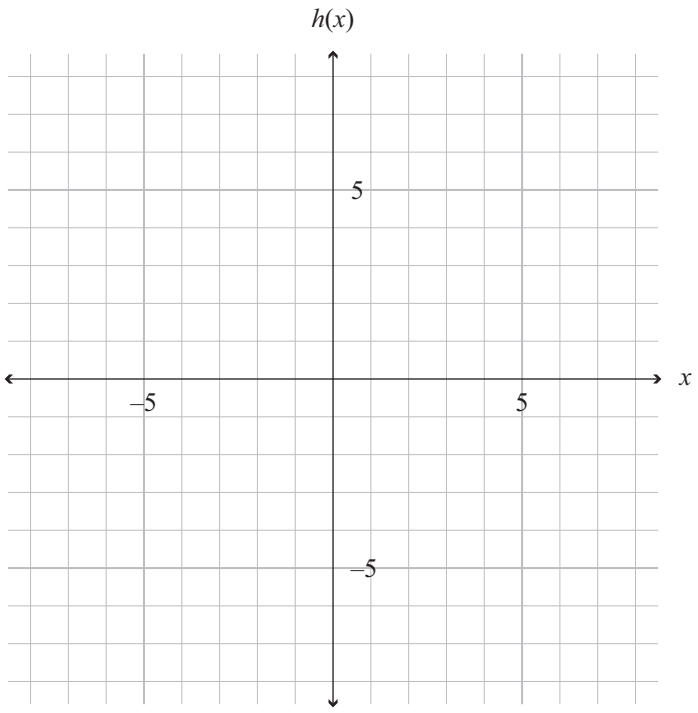
Find the rate of change of the area of the ripple, with respect to the radius, when the area is 49π m^2.

ASSESSOR'S USE ONLY

(d) Sketch the function $h(x)$ for the gradient function $h'(x)$ below, given that the maximum value of h is 5.

Show the vertex clearly.





If you need to redraw this graph, use the grid on page 15

ASSESSOR'S USE ONLY

(e) The gradient of a curve is given by $\frac{dy}{dx} = 6x^2 - 12x$.

The y-coordinate of the minimum turning point of the curve is 10.

Find the equation of the curve.

(f) A car is travelling at a constant speed until the car's brakes are applied.

The car's speed changes at a rate given by $-0.08t$ metres sec^{-2} after the brakes are applied, where t sec is the time since the brakes were applied.

3 seconds after the brakes are applied, the speed of the car is 5 metres sec^{-1}.

How far will the car travel with the brakes applied before it stops?

ASSESSOR'S USE ONLY

QUESTION THREE

ASSESSOR'S USE ONLY

(a) A curve $y = f(x)$ passes through (0,0) and has gradient function

$f'(x) = 4x + 3$.

Find the coordinates of the point on the curve where $x = -3$.

(b) (i) Find the x-coordinate of the point on the graph of $g(x) = 0.5x^2 - 5x$ where the gradient is equal to 2.

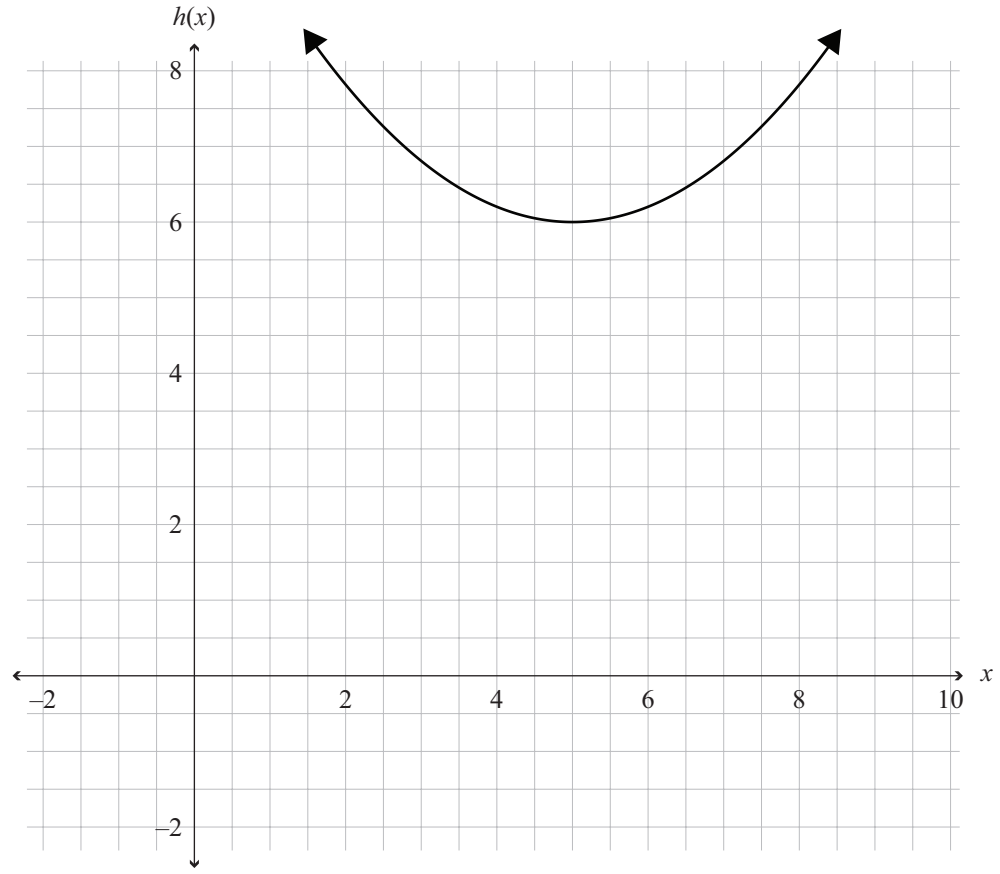
(ii) Find the equation of the tangent to the curve $g(x) = 0.5x^2 - 5x$ at the point (8,–8).

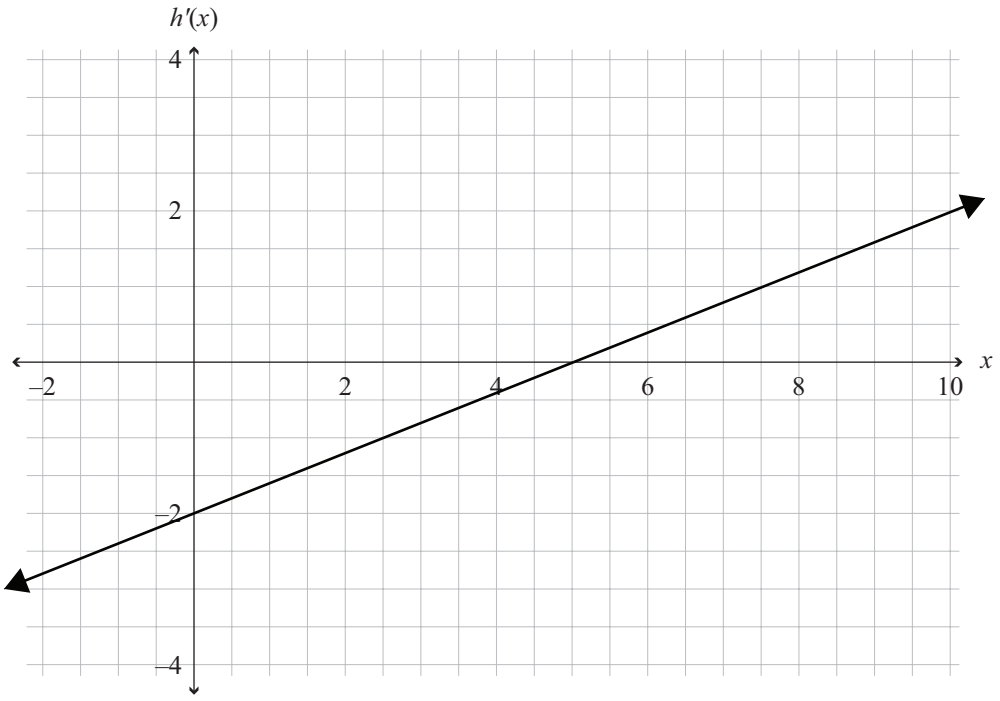
This page has been deliberately left blank.

The examination continues on the following page.

ASSESSOR'S USE ONLY

(c) The graph of the function $h(x)$ together with that of its gradient function are given below.





Find the equation of $h(x)$.

You must **use calculus methods** to obtain your answer.

ASSESSOR'S USE ONLY

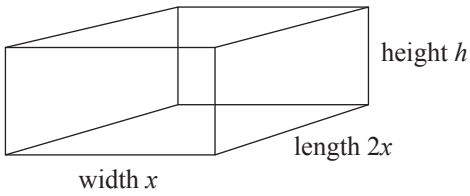
(d) The curve of $f(x) = Px^2 + Qx + 2$ has a turning point when $x = \frac{2}{3}$.

The curve passes through the point (1,9).

Find the coordinates of the point on the curve where $x = 3$.

(e) The frame of a crate is made up of 12 steel rods that have a total length of L cm.

The length of the crate is twice the width.

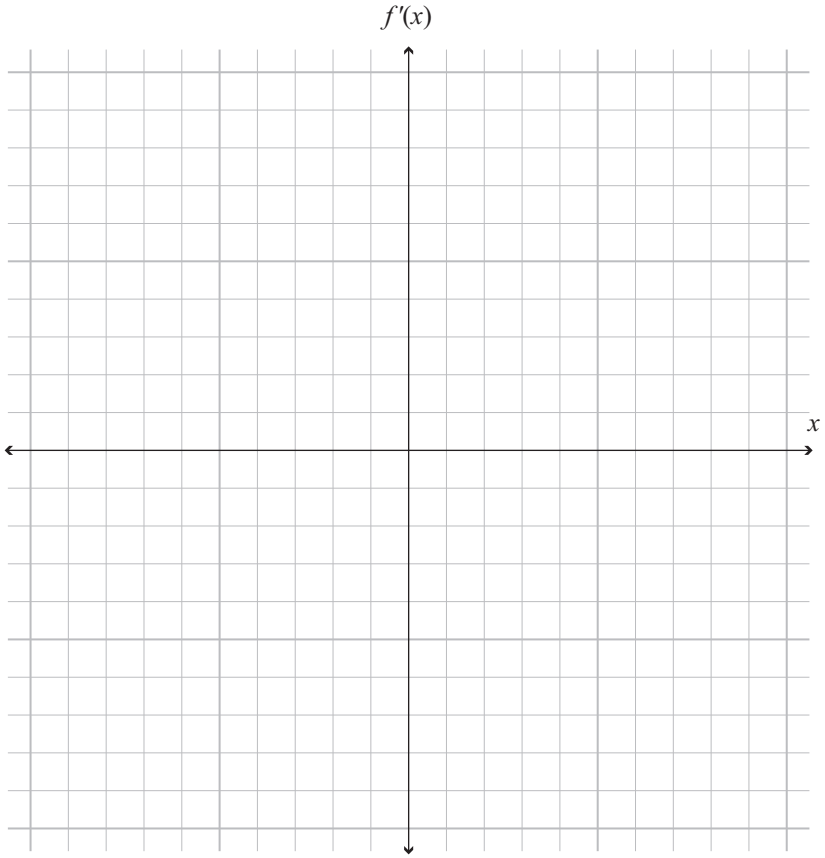


Show that the length of the crate will be $\frac{L}{9}$ cm when the volume is a maximum.

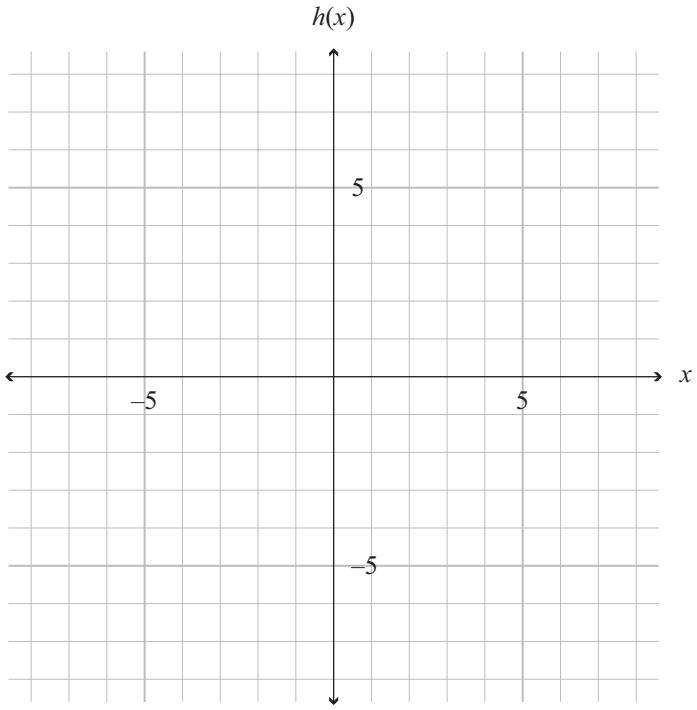
ASSESSOR'S USE ONLY

ASSESSOR'S USE ONLY

If you need to redraw your graph from Question Two (a), draw it on the grid below. Make sure it is clear which graph you want marked.



If you need to redraw your graph from Question Two (d), draw it on the grid below. Make sure it is clear which graph you want marked.



Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

91262